

# Large non-Gaussianities from DBI Galileon and resolution of sensitivity problem

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We have studied primordial non-Gaussian features through bispectrum and trispectrum analysis from a model of *potential driven DBI Galileon inflation* originating from background supergravity and Gauss-Bonnet terms. We have explicitly shown the violation of the widely accepted *Maldacena theorem* and *standard Suyama-Yamaguchi relation* in squeezed limit configuration which leads to the resolution of the well-known sensitivity problem between the non-Gaussian parameters ( $f_{NL}, \tau_{NL}, g_{NL}$ ) and tensor to scalar ratio ( $r$ ). Our analysis thus overcomes a generic drawback of the wide class of DBI inflationary models which was, of late, facing tension from observational ground. Hence large primordial non-Gaussianities can be obtained even from single field DBI Galileon and hence these class of models can be directly confronted with the forthcoming results of PLANCK.

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## I. INTRODUCTION

The physics of the early universe is a very rich area of theoretical physics, for there is a plethora of potential models that solve, at least partially, the well-known problems of the standard cosmological paradigm. Inflationary cosmology is the most successful branch which addressed all of these problems meticulously. This can however be explained by several class of models originated from a proper field theoretic or particle physics framework. But from observational point view a big issue may crop up in model discrimination and also in the removal of the degeneracy of cosmological parameters obtained from Cosmic Microwave Background (CMB) observations [1, 2]. In this context the study of primordial non-Gaussian feature acts as a powerful computational tool to discriminate and to rule out several class of proposed inflationary models. In the very recent days the analysis of bispectrum and trispectrum derived from the study of primordial features of non-Gaussianity [3–18] from different models of inflation has thus become an intriguing aspect in the context of inflationary model building as well as studies of CMB physics.

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The DBI inflationary model [19], [20] is one of the most interesting possibilities to realize large non-Gaussianity of the cosmic microwave background (CMB) temperature fluctuations. But this class of models has a serious drawback, commonly known as generic *sensitivity problem* which states that the non-Gaussian parameters derived from bispectrum and trispectrum analysis is highly sensitive to the tensor to scalar ratio [21–24]. This results in a tension between these two parameters from observational ground. To be specific, the parameter space of non-Gaussianity and tensor to scalar ratio as obtained from DBI models contradicts the future predictions of PLANCK [2] which has already started giving early results – the science run results are soon to follow. Very recently, a natural extension to the wide class of DBI models has been brought forward by tagging Galileon with the good old DBI model, resulting in so-called “DBI Galileon” [25–27] which, amidst its successes, results in unwanted degrees of freedom like ghosts, Laplacian and Tachyonic instabilities. In a recent paper [28] the present authors have demonstrated how these unwanted debris can be removed in spite of keeping all the good features of [25–27] intact by proposing DBI Galileon in D3 brane in the background of  $\mathcal{N}=1, \mathcal{D}=4$  SUGRA derived from D4 brane in  $\mathcal{N}=2, \mathcal{D}=5$  bulk SUGRA background. Starting from this model, In the present paper, our prime objective is to carry forward this rich structure of DBI Galileon as proposed in [28] in order to resolve the above mentioned *sensitivity problem* by studying bispectrum and trispectrum calculated from three and four point correlation function originated through cosmological perturbation, working upon third and fourth order perturbative action in *De Sitter (DS)* and *Beyond De Sitter (BDS)* limit. Subsequently, we demonstrate that it is indeed possible to have a parameter space for both non-Gaussianity and tensor-to-scalar ratio ( $r$ ) consistent with observational predictions of PLANCK.

The plan of the paper is as follows. First we exhaustively study primordial non-Gaussian features from the third order perturbative action through the non-linear parameter  $f_{NL}$  calculated from bispectrum (in non-local and equilateral limit configuration) including all possible scalar - tensor type of interactions in the different polarizing modes. Hence from the fourth order perturbative action we derive the expression for other two non-linear parameters  $g_{NL}$  and  $\tau_{NL}$  through trispectrum analysis. Finally, we have explicitly shown that the violation of the well known *Maldacena theorem* [29], [30] and *standard Suyama-Yamaguchi relation* [31], [32] in the squeezed limit configuration leading to the solution of the *sensitivity problem* of non-Gaussian parameters.

## II. THE BACKGROUND MODEL AND PERTURBATIVE ACTION

For systematic development of the formalism, let us briefly review from our previous paper [28] how one can construct the effective 4D inflationary potential for DBI Galileon starting from  $\mathcal{N} = 2, \mathcal{D} = 5$  SUGRA along with Gauss-Bonnet correction in the bulk geometry and D4 brane setup leads to an effective  $\mathcal{N} = 1, \mathcal{D} = 4$  SUGRA in the D3 brane. Here the total five dimensional model is described by the following action

$$S_{Total}^{(5)} = S_{EH}^{(5)} + S_{GB}^{(5)} + S_{D4\ brane}^{(5)} + S_{BulkSugra}^{(5)} \quad (2.1)$$

where

$$S_{EH}^{(5)} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g^{(5)}} [R_{(5)} - 2\Lambda_5], \quad (2.2)$$

$$S_{GB}^{(5)} = \frac{\alpha_{(5)}}{2\kappa_5^2} \int d^5x \sqrt{-g^{(5)}} [R^{ABCD(5)} R_{ABCD}^{(5)} - 4R^{AB(5)} R_{AB}^{(5)} + R_{(5)}^2]. \quad (2.3)$$

The D4 brane can be decomposed into two parts as

$$S_{D4\ brane}^{(5)} = S_{DBI}^{(5)} + S_{WZ}^{(5)}, \quad (2.4)$$

where the *DBI* action and the *Wess-Zumino* action is given by

$$S_{DBI}^{(5)} = -\frac{T_4}{2} \int d^5x \exp(-\Phi) \sqrt{-(\gamma^{(5)} + B^{(5)} + 2\pi\alpha' F^{(5)})}, \quad (2.5)$$

$$\begin{aligned} S_{WZ}^{(5)} = & -\frac{T_4}{2} \int \sum_{n=0,2,4} \hat{C}_n \wedge \exp(\hat{B}_2 + 2\pi\alpha' F_2) |_{4\ form} \\ = & \frac{1}{2} \int d^5x \sqrt{-g^{(5)}} \left\{ \epsilon^{ABCD} \left[ \partial_A \Phi^I \partial_B \Phi^J \left( \frac{C_{IJ} B_{KL}}{4T_4} \partial_C \Phi^K \partial_D \Phi^L + \frac{\pi\alpha' C_{IJ} F_{CD}}{2} \right. \right. \right. \\ & \left. \left. + \frac{C_0}{8T_4} B_{IJ} B_{KL} \partial_C \Phi^K \partial_D \Phi^L + \frac{\pi\alpha' C_0}{2} B_{IJ} F_{CD} \right) + 2\pi^2 \alpha'^2 T_4 C_0 F_{AB} F_{CD} - T_4 \nu(\Phi) \right] \right\} \end{aligned} \quad (2.6)$$

where  $T_{(4)}$  is the D4 brane tension,  $\alpha'$  is the Regge Slope determines length scale of the string,  $\exp(-\Phi)$  is the closed string dilaton and  $C_0$  is the Axion. In eqn(2.1)  $\mathcal{N}=2, \mathcal{D}=5$  bulk supergravity action can be written as

$$S_{Bulk\ Sugra}^{(5)} = \frac{1}{2} \int d^5x \sqrt{-g_{(5)}} e_{(5)} \left[ -\frac{M_s^3 R^{(5)}}{2} + \frac{i}{2} \bar{\Psi}_{i\tilde{m}} \Gamma^{\tilde{m}\tilde{n}\tilde{q}} \nabla_{\tilde{n}} \Psi_{\tilde{q}}^i - S_{IJ} F_{\tilde{m}\tilde{n}}^I F^{I\tilde{m}\tilde{n}} - \frac{1}{2} g_{\alpha\beta} (D_{\tilde{m}} \phi^\mu) (D^{\tilde{m}} \phi^\nu) \right. \\ \left. + \text{Fermionic} + \text{Chern} - \text{Simons} + \text{Pauli mass} \right], \quad (2.7)$$

In the present context 5-dimensional coordinates  $X^A = (x^\alpha, y)$ , where  $y$  parameterizes the extra dimension compactified on the closed interval  $[-\pi R, +\pi R]$  and  $Z_2$  orbifolding symmetry is imposed.

Applying dimensional reduction technique the total effective model for  $D3\ DBI\ Galileon$  in background  $\mathcal{N}=1, \mathcal{D}=4$  SUGRA is described by the following action:

$$S_{(4)} = \int d^4x \sqrt{-g^{(4)}} \left[ \hat{K}(\phi, \tilde{X}) - \tilde{G}(\phi, \tilde{X}) \square^{(4)} \phi + \tilde{l}_1 R_{(4)} \right. \\ \left. + \tilde{l}_4 \left( \mathcal{C}(1) R^{\alpha\beta\gamma\delta(4)} R_{\alpha\beta\gamma\delta}^{(4)} - 4\mathcal{I}(2) R^{\alpha\beta(4)} R_{\alpha\beta}^{(4)} + \mathcal{A}(6) R_{(4)}^2 \right) + \tilde{l}_3 \right], \quad (2.8)$$

where

$$\hat{K}(\phi, \tilde{X}) = -\frac{\tilde{D}}{f(\phi)} \left[ \sqrt{1 - 2Q\tilde{X}\tilde{f}} - Q_1 \right] - \tilde{C}_5 \tilde{G}(\tilde{\phi}, \tilde{X}) - 2X\tilde{M}(T, T^\dagger) - V(\phi), \\ \tilde{l}_1 = \left\{ \frac{1}{2\kappa_{(4)}^2} \left[ 1 + \frac{\alpha_{(4)}}{R^2\beta^2} (24\mathcal{I}(2) - 24\mathcal{A}(9) - 16\mathcal{A}(10)) \right] - \frac{\alpha_{(4)}\mathcal{C}(2)}{\kappa_{(4)}^2 R^2\beta^2} \right\}, \tilde{l}_4 = \frac{\alpha_{(4)}}{2\kappa_{(4)}^2}, \\ \tilde{l}_3 = \frac{1}{2\kappa_{(4)}^2} \left[ \frac{\alpha_{(4)}}{R^4\beta^4} (24\mathcal{C}(24) - 144\mathcal{I}(4) - 64\mathcal{A}(5) + 144\mathcal{A}(7) + 64\mathcal{A}(8) + 192\mathcal{A}(11)) - \frac{3M_s^3\beta b_0^6}{2\kappa_{(4)}^2 M_{PL}^2 R^5} \mathcal{I}(1) \right] \quad (2.9)$$

All the constants appearing in equation(2.8) and equation(2.9) are explicitly mentioned in [28]. The one-loop corrected Coleman Weinberg potential is given by:

$$V(\phi) = \sum_{m=-2, m \neq -1}^2 C_{2m} \left[ 1 + D_{2m} \ln \left( \frac{\phi}{M} \right) \right] \phi^{2m}, \quad (2.10)$$

where  $C_0 = (T_3 \tilde{v}_0 + \beta R \mathcal{I}(2) \tilde{f}_0 + \tilde{Z}(T, T^\dagger) \mathcal{A}(13) v^4)$ ,  $C_{-4} = T_3 \tilde{v}_4$ ,  $C_2 = (\beta R \mathcal{I}(2) \tilde{f}_2 - g v^2 \tilde{Z}(T, T^\dagger) \mathcal{A}(13))$ ,  $C_4 = (\beta R \mathcal{I}(2) \tilde{f}_4 + \frac{\tilde{Z}(T, T^\dagger) \mathcal{A}(13) g^2}{4})$  and  $D_0 = 0$ . Here the energy scale of inflation has a GUT scale window  $0.658 \times 10^{16} GeV < \sqrt[4]{C_0} < 0.667 \times 10^{16} GeV$ .

Using this model our next goal is to exhaustively study the primordial features of non-Gaussianities by deriving the explicit form of bispectrum and trispectrum in  $DS$  and  $BDS$  limit which can solve the generic problem of well known DBI inflation in our framework. Moreover the explicit calculation confirms that effective DBI Galileon in D3 brane including the Gauss-Bonnet correction is necessary ingredient to resolve the sensitivity problem of DBI inflation. To serve this purpose we begin with our discussion to the following perturbative action which can be expressed as:

$$S = \int dt d^3x \left( \underbrace{\mathcal{L}_{\zeta\zeta} + \mathcal{L}_{hh} + \mathcal{L}_{vv}}_{\text{power spectrum contribution}} + \underbrace{\mathcal{L}_{\zeta\zeta\zeta} + \mathcal{L}_{\zeta hh} + \mathcal{L}_{\zeta\zeta h} + \mathcal{L}_{hhh} + \mathcal{L}_{hhv} + \mathcal{L}_{hvv} + \mathcal{L}_{vvv} + \mathcal{L}_{\zeta\zeta v} + \mathcal{L}_{\zeta v v} + \mathcal{L}_{\zeta hv}}_{\text{bispectrum contribution}} \right. \\ \left. + \underbrace{\mathcal{L}_{\zeta\zeta\zeta\zeta} + \mathcal{L}_{\zeta\zeta\zeta h} + \mathcal{L}_{\zeta\zeta hh} + \mathcal{L}_{\zeta hhh} + \mathcal{L}_{hhhh} + \mathcal{L}_{hhhv} + \mathcal{L}_{hhvv} + \mathcal{L}_{hvvv} + \mathcal{L}_{vvvv} + \mathcal{L}_{vvv\zeta} + \mathcal{L}_{vv\zeta\zeta} + \mathcal{L}_{v\zeta\zeta\zeta} + \mathcal{L}_{\zeta hvv} + \mathcal{L}_{\zeta\zeta hv} + \mathcal{L}_{\zeta h hv}}_{\text{trispectrum contribution}} \right) \quad (2.11)$$

where  $\mathcal{L}_{ab}$ ,  $\mathcal{L}_{abc}$  and  $\mathcal{L}_{abcd}$  represent second, third and fourth order Lagrangian density respectively with  $(a, b, c, d := \zeta, v, h)$ . In equation(2.11)  $\zeta$ ,  $v$  and  $h$  stands for the primordial curvature (scalar), vector and tensor perturbation modes. It is imperative to mention that, in this context, we neglect all the contributions from vector modes in all order of perturbation theory. Further, we neglect the contribution of tensor modes in the fourth order perturbative action as well as in the four point correlation functions. To explore the non-Gaussian features from our model we start with the following generalized ansatz for the primordial curvature perturbation and tensor perturbation:

$$\mathcal{X}^m(\vec{x}) = \mathcal{X}_G^m(\vec{x}) + \frac{3}{81} (f_{NL}^{local})^{abm} \{ (\mathcal{X}_G(\vec{x}))_a (\mathcal{X}_G(\vec{x}))_b - \langle (\mathcal{X}_G(\vec{x}))_a (\mathcal{X}_G(\vec{x}))_b \rangle \} + \frac{9}{25} (g_{NL}^{local})^{abcm} (\mathcal{X}_G(\vec{x}))_a (\mathcal{X}_G(\vec{x}))_b (\mathcal{X}_G(\vec{x}))_c \\ + \frac{81}{125} (h_{NL}^{local})^{abcdm} \{ (\mathcal{X}_G(\vec{x}))_a (\mathcal{X}_G(\vec{x}))_b (\mathcal{X}_G(\vec{x}))_c (\mathcal{X}_G(\vec{x}))_d - \langle (\mathcal{X}_G(\vec{x}))_a (\mathcal{X}_G(\vec{x}))_b (\mathcal{X}_G(\vec{x}))_c (\mathcal{X}_G(\vec{x}))_d \rangle \} \\ + \dots - \langle \mathcal{X}(\vec{x}) \rangle \quad (2.12)$$

where  $(a, b, c, d = \zeta, h)$ . Specifically  $(\mathcal{X}_G(\vec{x}))_h = \int \frac{d^3 k}{(2\pi)^3} \left( \mathcal{X}_G(\vec{k}) \right)_h \exp(i\vec{k} \cdot \vec{x})$  where  $\left( \mathcal{X}_G(\vec{k}) \right)_h = \bigoplus^\lambda(\vec{k}) = h_{ij}(\vec{k}) e_{ij}^{\dagger(\lambda)}(\vec{k})$ . Moreover in this context the n-point correlation functions are given by,

$$\langle (\mathcal{X}_G(\vec{x}))_a (\mathcal{X}_G(\vec{x}))_b (\mathcal{X}_G(\vec{x}))_c (\mathcal{X}_G(\vec{x}))_d \dots n \text{ terms} \rangle = \begin{cases} 0 & : n=1, 3, 5, \dots \forall (a, b, c, d) \\ \text{finite} (\neq 0) & : n=4, 6, 8, \dots \forall (a, b, c, d) \text{ and } n=2 \text{ with } a \neq b \\ 0 & : n=2 \text{ with } a=b. \end{cases} \quad (2.13)$$

### III. TREE LEVEL BISPECTRUM ANALYSIS

#### A. Three scalar correlation

To calculate the scalar bispectrum for D3 DBI Galileon we consider here the third order perturbative action up to total derivatives. Using the uniform field gauge analysis the third order perturbative action for three scalar interaction can be written as:

$$\begin{aligned} (S^{(4)})_{\zeta\zeta\zeta} = \int dt d^3 x \left\{ a^3 \bar{C}_1 M_{PL}^2 \zeta \dot{\zeta}^2 + a \bar{C}_2 M_{PL}^2 \zeta (\partial \zeta)^2 + a^3 \bar{C}_3 M_{PL} \dot{\zeta}^3 + a^3 \bar{C}_4 \dot{\zeta} (\partial_i \zeta) (\partial_i \tilde{\chi}) + a^3 \left( \frac{\bar{C}_5}{M_{PL}^2} \right) \partial^2 \zeta (\partial \tilde{\chi})^2 \right. \\ \left. + a \bar{C}_6 \dot{\zeta}^2 \partial^2 \zeta + \left( \frac{\bar{C}_7}{a} \right) [\partial^2 \zeta (\partial \zeta)^2 - \zeta \partial_i \partial_j (\partial_i \zeta) (\partial_j \zeta)] + a \frac{\bar{C}_8}{M_{PL}} [\partial^2 \zeta \partial_i \zeta \partial_i \tilde{\chi} - \zeta \partial_i \partial_j (\partial_i \zeta) (\partial_j \tilde{\chi})] + \mathcal{R} \frac{\delta \mathcal{L}_2}{\delta \zeta} \Big|_1 \right\}, \end{aligned} \quad (3.1)$$

where

$$\frac{\delta \mathcal{L}_2}{\delta \zeta} \Big|_1 = -2 \left[ \frac{d}{dt} (a^3 Y_S \dot{\zeta}) - a Y_S c_s^2 \partial^2 \zeta \right] \quad (3.2)$$

can be calculated from the second order action [28]

$$(S^{(4)})_{\zeta\zeta} = \int dt d^3 x a^3 Y_S \left[ \dot{\zeta}^2 - \frac{c_s^2}{a^2} (\partial \zeta)^2 \right]. \quad (3.3)$$

Here  $\bar{C}_i (i = 1, 2, 3, \dots, 8)$  are dimensionless co-efficients defined as:

$$\begin{aligned} \bar{C}_1 = \frac{Y_S}{M_{PL}^2} \left[ 3 - \frac{L_1 H}{c_s^2} \left( 3 + \frac{\dot{Y}_S}{H Y_S} \right) + \frac{d}{dt} \left( \frac{L_1}{c_s^2} \right) \right], \bar{C}_2 = \left[ 1 + \frac{1}{a} \frac{d}{dt} (a L_1 \{Y_S - t_1\}) \right], \\ \bar{C}_3 = \frac{L_1}{M_{PL}} \left[ L_1 (L_1 a_1 + a_3) + a_{12} + (a_9 + L_1 a_4) \frac{Y_S}{t_1} + \frac{Y_S}{c_s^2} \right], \bar{C}_4 = -\frac{Y_S}{2t_1} \left\{ 1 + 2t_1 \left[ \frac{d}{dt} \left( \frac{A_5}{t_1^2} \right) - \frac{3HA_5}{t_1^2} \right] \right\}, \\ \bar{C}_5 = \frac{M_{PL}^2}{2t_1^2} \left[ \frac{3M_{PL}^2}{2} (1 - HL_1) \right] - \frac{M_{PL}^2}{2} \frac{d}{dt} \left( \frac{A_5}{t_1^2} \right), \bar{C}_6 = L_1^2 [2M_{PL}^2 - L_1 a_4], \\ \bar{C}_7 = \frac{L_1^2 M_{PL}^2 (1 - HL_1)}{6} - \frac{c_s^2 Y_S L_1^2 M_{PL}^2}{2t_1} + \frac{M_{PL}^2}{6} \frac{d}{dt} (L_1^3), \bar{C}_8 = M_{PL} \left\{ \frac{L_1 M_{PL}^2}{t_1} (HL_1 - 1) + \frac{c_s^2 Y_S L_1 M_{PL}^2}{t_1^2} \right\} \end{aligned} \quad (3.4)$$

and the co-efficient of  $\frac{\delta \mathcal{L}_2}{\delta \zeta} \Big|_1$  involves spatial and time derivative in equation(5.9) is defined by the following expression:

$$\mathcal{R} = \frac{A_5}{t_1^2} \{ (\partial_k \zeta) (\partial_k \tilde{\chi}) - \partial^{-2} \partial_i \partial_j [(\partial_i \zeta) (\partial_j \tilde{\chi})] \} + p_1 \zeta \dot{\zeta} - \frac{A_5 L_1}{2t_1 a^2} \{ (\partial \zeta)^2 - \partial^{-2} \partial_i \partial_j [(\partial_i \zeta) (\partial_j \zeta)] \}. \quad (3.5)$$

In this context  $A_3 = 2Y_S$ ,  $A_5 = -\frac{L_1 M_{PL}^2}{2}$  and  $\mathcal{R} \rightarrow 0$  as  $k \rightarrow 0$  at large scale. Throughout our analysis we use the Hubble parameter as  $H = \left[ \frac{\Lambda_{(4)} + 8\pi G_{(4)} V(\phi)}{\bar{g}_1} \right]^{\frac{1}{4}}$  from our previous paper [28]. Additionally  $L_1 = \left( \frac{M_{PL}^2}{H M_{PL}^2 - \phi X \bar{G}_X} \right) = -p_1 c_s^2$  and  $\tilde{\chi} = \partial^{-2} (Y_S \dot{\zeta})$ . Now following the prescription of *in-in formalism* in the interacting picture the *three point correlation function* both for *DS* and *BDS*, after some trivial algebra took:

$$\begin{aligned} \langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3) \rangle &= -i \sum_{j=1}^8 \int_{-\infty}^0 d\eta \, a \, \langle 0 | \left[ \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3), \left( H_{int}^{(j)}(\eta) \right)_{\zeta\zeta\zeta} \right] | 0 \rangle \\ &= (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \mathcal{B}_{\zeta\zeta\zeta}(\vec{k}_1, \vec{k}_2, \vec{k}_3), \end{aligned} \quad (3.6)$$

where the total Hamiltonian in the interaction picture can be expressed in terms of the third order Lagrangian density as  $(H_{int}(\eta))_{\zeta\zeta\zeta} = \sum_{j=1}^8 \left( H_{int}^{(j)}(\eta) \right)_{\zeta\zeta\zeta} = - \int d^3x (\mathcal{L}_3)_{\zeta\zeta\zeta}$ . Moreover the *bispectrum*  $\mathcal{B}_{\zeta\zeta\zeta}(\vec{k}_1, \vec{k}_2, \vec{k}_3)$  is defined as:

$$\mathcal{B}_{\zeta\zeta\zeta}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{(2\pi)^4 \mathcal{P}_\zeta^2}{\prod_{i=1}^3 k_i^3} \mathcal{A}_{\zeta\zeta\zeta}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{6}{5} f_{NL;1}^{local} P_\zeta^2 \quad (3.7)$$

where the symbol ;1 is used for three scalar correlation. Here  $\mathcal{A}_{\zeta\zeta\zeta}(\vec{k}_1, \vec{k}_2, \vec{k}_3)$  is the *shape function* for bispectrum and  $P_\zeta^2$  is used for normalization of E-mode polarization expressed in terms of the new combination of the cyclic permutations of two-point correlation functions given by

$$P_\zeta^2 = P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1). \quad (3.8)$$

In this context  $f_{NL}^{local}$  represents the non linear parameter carrying the signature of primordial non-Gaussianities of the curvature perturbation in bispectrum. The explicit form of  $f_{NL}^{local}$  characterizing the bispectrum can be expressed as:

$$\begin{aligned} [f_{NL;1}^{local}]_{DS} = & \frac{10M_{PL}^2}{3Y_S} \frac{1}{\sum_{i=1}^3 k_i^3} \left\{ \frac{\bar{C}_1}{4} \left( \frac{2}{K} \sum_{i,j(i>j)=1}^3 k_i^2 k_j^2 - \frac{1}{K^2} \sum_{i,j(i \neq j)=1}^3 k_i^2 k_j^3 \right) + \frac{\bar{C}_2}{4c_s^2} \left( \frac{1}{2} \sum_{i=1}^3 k_i^3 \right. \right. \\ & + \frac{2}{K} \sum_{i,j(i>j)=1}^3 k_j^2 k_j^2 - \frac{1}{K^2} \sum_{i,j(i \neq j)=1}^3 k_i^3 k_j^3 \Big) + \frac{3H\bar{C}_3}{2M_{PL}} \frac{(k_1 k_2 k_3)^2}{K^3} + \frac{Y_S \bar{C}_4}{8M_{PL}^2} \left( \sum_i k_i^3 \right. \\ & - \frac{1}{2} \sum_{i,j(i>j)=1}^3 k_j^2 k_j^2 - \frac{2}{K^2} \sum_{i,j(i \neq j)=1}^3 k_i^3 k_j^3 \Big) + \frac{\bar{C}_5 Y_S^2}{4M_{PL}^2 K^2} \left[ \sum_{i=1}^3 k_i^5 + \frac{1}{2} \sum_{i,j(i \neq j)=1}^3 k_i k_j^4 \right. \\ & - \frac{3}{2} \sum_{i,j(i \neq j)=1}^3 k_i^2 k_j^3 - k_1 k_2 k_3 \sum_{i,j(i>j)=1}^3 k_i k_j \Big] + \frac{3\bar{C}_6 H^2}{c_s^2 M_{PL}^2} \frac{(k_1 k_2 k_3)^2}{K^3} + \frac{\bar{C}_7 H^2}{2c_s^2 M_{PL}^2 K} \\ & \times \left( 1 + \frac{1}{K^2} \sum_{i,j(i>j)=1}^3 k_i k_j + \frac{3k_1 k_2 k_3}{K^3} \right) \left[ \frac{3}{4} \sum_{i=1}^3 k_i^4 - \frac{3}{2} \sum_{i,j(i>j)=1}^3 k_i^2 k_j^2 \right] \\ & \left. + \frac{\bar{C}_8 H Y_S}{8c_s^2 M_{PL}^2 K^2} \left[ \frac{3}{2} k_1 k_2 k_3 \sum_{i=1}^3 k_i^2 - \frac{5}{2} k_1 k_2 k_3 K^2 - 6 \sum_{i,j(i \neq j)=1}^3 k_i^2 k_j^3 - \sum_{i=1}^3 k_i^5 + \frac{7K}{2} \sum_{i=1}^3 k_i^4 \right] \right\} \quad (3.9) \end{aligned}$$

$$\begin{aligned} [f_{NL;1}^{local}]_{BDS} = & \frac{10}{3 \sum_{i=1}^3 k_i^3} \left( \frac{k_1 k_2 k_3}{2K^3} \right)^{n_\zeta - 1} \left| \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right|^2 \left\{ \bar{C}_1 \left[ \frac{3}{4} \mathcal{I}_1(n_\zeta - 1) - \frac{3 - \epsilon_V}{4c_s^2} \left( \frac{1 + Y_S}{1 + \epsilon_V} \right)^2 \mathcal{I}_1(\tilde{\nu}) \right] \right. \\ & + \frac{3(1 - \epsilon_V - s_V^S)}{2Y_S} \left[ \mathcal{F}_3 \mathcal{I}_3(n_\zeta - 1) + \frac{\mathcal{E}_3}{c_s^2} \mathcal{I}_3(\tilde{\nu}) \right] + \frac{\bar{C}_4}{8} \mathcal{I}_4(\tilde{\nu}) + \frac{\bar{C}_5 Y_S}{4c_s^2} \mathcal{I}_5(\tilde{\nu}) \\ & \left. + \frac{3(1 - \epsilon_V - s_V^S)^2}{Y_S} \left[ \mathcal{F}_6 \mathcal{I}_6(n_\zeta - 1) + \frac{\mathcal{E}_6}{c_s^2} \mathcal{I}_6(\tilde{\nu}) \right] + \frac{\bar{C}_7 (1 - \epsilon_V - s_V^S)^2}{2Y_S c_s^2} \mathcal{I}_7(\tilde{\nu}) + \frac{\bar{C}_8 (1 - \epsilon_V - s_V^S)}{8c_s^2} \mathcal{I}_8(\tilde{\nu}) \right\}. \quad (3.10) \end{aligned}$$

where

$$\begin{aligned} \mathcal{I}_1(x) = & \cos \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \Gamma(1+x) \left[ \frac{2+x}{K} \sum_{i>j} k_i^2 k_j^2 - \frac{1+x}{K^2} \sum_{i \neq j} k_i^2 k_j^3 \right], \mathcal{I}_6(x) = \frac{(k_1 k_2 k_3)^2}{K^3} \cos \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \frac{(6+x)\Gamma(3+x)}{12}, \\ \mathcal{I}_2(x) = & \cos \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \Gamma(1+x) \left[ \frac{K}{1-x} - \frac{1}{K} \sum_{i>j} k_i k_j - \frac{1+x}{K^2} k_1 k_2 k_3 \right], \mathcal{I}_3(x) = \frac{(k_1 k_2 k_3)^3}{K^3} \frac{\Gamma(3+x)}{2} \cos \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right), \\ \mathcal{I}_4(x) = & \cos \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \left\{ \frac{(\vec{k}_1 \cdot \vec{k}_2) k_3^2}{K} \left[ (3+x)\Gamma(1+x) - \Gamma(2+x) \frac{k_3}{K} \right] + \frac{(\vec{k}_2 \cdot \vec{k}_3) k_1^2}{K} \left[ (3+x)\Gamma(1+x) - \Gamma(2+x) \frac{k_1}{K} \right] \right. \\ & \left. + \frac{(\vec{k}_3 \cdot \vec{k}_1) k_2^2}{K} \left[ (3+x)\Gamma(1+x) - \Gamma(2+x) \frac{k_2}{K} \right] \right\}, \\ \mathcal{I}_5(x) = & \cos \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \left\{ \frac{(\vec{k}_1 \cdot \vec{k}_2) k_3^2}{K} \left[ \Gamma(1+x) + \Gamma(2+x) \frac{k_3}{K} \right] + \frac{(\vec{k}_2 \cdot \vec{k}_3) k_1^2}{K} \left[ \Gamma(1+x) + \Gamma(2+x) \frac{k_1}{K} \right] + \right. \\ & \left. \frac{(\vec{k}_3 \cdot \vec{k}_1) k_2^2}{K} \left[ \Gamma(1+x) + \Gamma(2+x) \frac{k_2}{K} \right] \right\}, \\ \mathcal{I}_7(x) = & \cos \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \frac{2+x}{2} \left[ \Gamma(1+x) + \Gamma(2+x) \left( \frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{K^2} + (3+x) \frac{k_1 k_2 k_3}{K^3} \right) \right] \left\{ \frac{(\vec{k}_1 \cdot \vec{k}_2) k_3^2}{K} + \frac{(\vec{k}_2 \cdot \vec{k}_3) k_1^2}{K} + \frac{(\vec{k}_3 \cdot \vec{k}_1) k_2^2}{K} \right\}, \\ \mathcal{I}_8(x) = & \cos \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \left\{ \frac{(\vec{k}_1 \cdot \vec{k}_2) k_3^2}{K} \left[ (3+x)\Gamma(1+x) + (3+x)\Gamma(2+x) \frac{k_3}{K} - \Gamma(3+x) \frac{k_3^2}{K^2} \right] + \frac{(\vec{k}_2 \cdot \vec{k}_3) k_1^2}{K} \left[ (3+x)\Gamma(1+x) \right. \right. \\ & \left. \left. + (3+x)\Gamma(2+x) \frac{k_1}{K} - \Gamma(3+x) \frac{k_1^2}{K^2} \right] + \frac{(\vec{k}_3 \cdot \vec{k}_1) k_2^2}{K} \left[ (3+x)\Gamma(1+x) + (3+x)\Gamma(2+x) \frac{k_2}{K} - \Gamma(3+x) \frac{k_2^2}{K^2} \right] \right\}. \quad (3.11) \end{aligned}$$

Here we have defined  $K = k_1 + k_2 + k_3$ ,  $x = (n_\zeta - 1, \tilde{\nu})$  and

$$\begin{aligned}
\tilde{\nu} &:= \left( \frac{s_V^S - 2\epsilon_V}{1 - \epsilon_V - s_V^S} \right), \\
\mathcal{F}_3 &:= -\frac{Y_S(1 + Y_S)}{1 + \epsilon_V} \left[ 1 + 2\frac{Y_S - \epsilon_V + (1 + Y_S)\rho_3}{1 + \epsilon_V} + 2\mathcal{T}_3 \right], \\
\frac{\dot{\phi} X^2 \tilde{G}_{XX}}{H} &= \left( \rho_3 + \frac{\rho_4}{c_s^2} \right), \\
\frac{\nu_s}{\Sigma_G} &:= \left( \mathcal{T}_3 + \frac{\mathcal{T}_4}{c_s^2} \right), \\
\mathcal{E}_3 &:= -\frac{Y_S(1 + Y_S)}{1 + \epsilon_V} \left[ 2\mathcal{T}_4 - \frac{1 + Y_S}{1 + \epsilon_V} (1 - 2\rho_4) \right], \\
\mathcal{F}_6 &:= \frac{2(1 + Y_S)^3}{(1 + \epsilon_V)^3} \left[ \frac{Y_S - \epsilon_V}{1 + Y_S} + \rho_3 \right], \\
\mathcal{E}_6 &:= \frac{2\rho_4(1 + Y_S)^3}{(1 + \epsilon_V)^3}
\end{aligned} \tag{3.12}$$

with four new constants  $\rho_3, \rho_4, \mathcal{T}_3, \mathcal{T}_4$ . For the numerical estimation we have further used the *equilateral configuration* ( $k_1 = k_2 = k_3 = k$  and  $K = 3k$ ) in which the non-linear parameter  $f_{NL}$  can be simplified to the following form as:

$$\begin{aligned}
[f_{NL;1}^{equil}]_{DS} &= \left\{ \frac{85}{324} \left( 1 - \frac{1}{c_s^2} \right) + \frac{1}{c_s^2} \left( \frac{5}{12} \eta_s - \frac{85}{54} s_V^S + \frac{5}{4} \epsilon_s - \frac{305}{162} \delta_{GX} \right) - \frac{10\nu_G}{81\Sigma_G} + \delta_{GX} \left( \frac{20\lambda_G}{81\epsilon_s} + \frac{65}{162c_s^2\epsilon_s} \right. \right. \\
&\quad \left. \left. - \frac{10\eta_s}{27c_s^2} + \frac{10\epsilon_s}{27c_s^2} - \frac{4\sqrt{V'(\phi)}M_{PL}^2}{\sqrt{\mathcal{G}(\phi)}} + \frac{85}{108c_s^2} (\epsilon_s + \eta_s - 2s_V^S) - \frac{65}{72\epsilon_s} + \frac{5\epsilon_s}{54c_s^2} (3 - \epsilon_s) - \frac{5\epsilon_s^2}{108c_s^2} - \frac{10\nu_G}{81} \right) \right\} \\
&\tag{3.13}
\end{aligned}$$

$$\begin{aligned}
[f_{NL;1}^{equil}]_{BDS} &= \frac{10}{9k^3} \left( \frac{1}{54} \right)^{n_\zeta - 1} \left| \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right|^2 \left\{ \left( 3 \left( 1 - \frac{1}{c_s^2} \right) - \frac{Y_S \delta_V}{c_s^2} + \frac{Y_S^2}{c_s^2} - \frac{2Y_S s_V^S}{c_s^2} \right) \right. \\
&\quad \times \left[ \frac{3}{4} \mathcal{I}_1^{equil}(n_\zeta - 1) - \frac{3 - \epsilon_V}{4c_s^2} \left( \frac{1 + Y_S}{1 + \epsilon_V} \right)^2 \mathcal{I}_1^{equil}(\tilde{\nu}) \right] + \frac{3(1 - \epsilon_V - s_V^S)}{2Y_S} \left[ \mathcal{F}_3 \mathcal{I}_3^{equil}(n_\zeta - 1) + \frac{\mathcal{E}_3}{c_s^2} \mathcal{I}_3^{equil}(\tilde{\nu}) \right] \\
&\quad - \frac{1}{8} \left[ \frac{Y_S}{2} + \frac{Y_S}{2} (3 - Y_S) \right] \mathcal{I}_4^{equil}(\tilde{\nu}) + \frac{Y_S}{4c_s^2} \left( \frac{4\epsilon_V - Y_S(3 - \epsilon_V)}{4(1 + \epsilon_V)} \right) \mathcal{I}_5^{equil}(\tilde{\nu}) \\
&\quad + \frac{3(1 - \epsilon_V - s_V^S)^2}{Y_S} \left[ \mathcal{F}_6 \mathcal{I}_6^{equil}(n_\zeta - 1) + \frac{\mathcal{E}_6}{c_s^2} \mathcal{I}_6^{equil}(\tilde{\nu}) \right] - \frac{(1 - \epsilon_V - s_V^S)^2 (1 + Y_S)^2 (Y_S - \epsilon_V)}{2Y_S c_s^2 (1 + \epsilon_V)^3} \mathcal{I}_7^{equil}(\tilde{\nu}) \\
&\quad \left. + \frac{(1 + Y_S)(Y_S - \epsilon_V)(1 - \epsilon_V - s_V^S)}{4c_s^2 (1 + \epsilon_V)^2} \mathcal{I}_8^{equil}(\tilde{\nu}) \right\}. \\
&\tag{3.14}
\end{aligned}$$

In the equilateral configuration for *BDS*

$$\begin{aligned}
\mathcal{I}_1^{equil}(x) &= \mathcal{I}_5^{equil}(x) = \frac{\mathcal{I}_4^{equil}(x)}{2}, \\
\mathcal{I}_2^{equil}(x) &= \frac{3\mathcal{I}_8^{equil}(x)}{2(1 - x)}, \\
\mathcal{I}_6^{equil}(x) &= \left( 1 + \frac{x}{2} \right) \mathcal{I}_3^{equil}(x).
\end{aligned} \tag{3.15}$$

Now using the tensor-to-scalar ratio

$$[r(k)]_{DS} = \left( 16\epsilon_s c_s \left[ 1 - \frac{3}{2} \mathcal{O}(\epsilon_T^2) \right] \right)_* \tag{3.16}$$

$$[r(k)]_{BDS} = \left( 16 \cdot 2^{2(\nu_T - \nu_s)} \left| \frac{\Gamma(\nu_T)}{\Gamma(\nu_s)} \right|^2 \left( \frac{1 - \epsilon_V - s_V^T}{1 - \epsilon_V - s_V^S} \right)^2 c_s \epsilon_s \left[ 1 - \frac{3}{2} \mathcal{O}(\epsilon_T^2) \right] \right)_* \tag{3.17}$$

the sound speed  $c_s$  can be eliminated from the equation(3.13) and equation(3.14) also. Most significantly the numerical

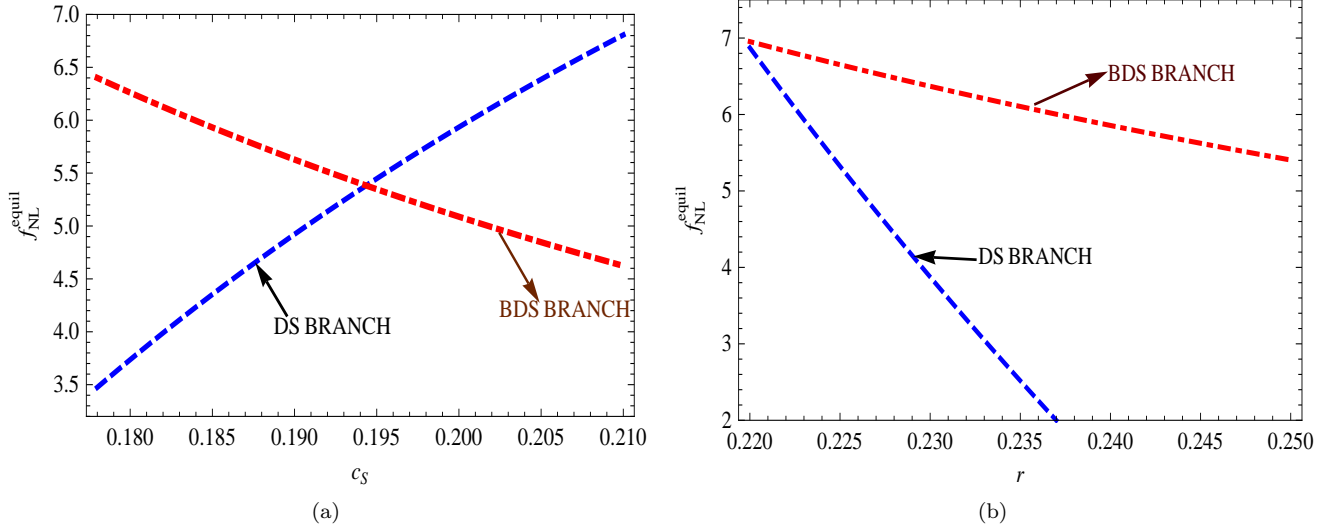


FIG. 1: (a) Variation of the  $f_{NL}$  vs sound speed for scalar modes  $c_s$ , (b) Variation of the  $f_{NL}$  vs tensor to scalar ratio  $r$  in  $DS$  and  $BDS$  limit.

value of  $f_{NL;1}^{equil}$  in the equilateral limit is obtained from our set up as  $2.8 < f_{NL;1}^{equil} < 7$  in  $DS$  limit and  $4 < f_{NL;1}^{equil} < 7$  in  $BDS$  limit within the window for tensor-to-scalar ratio  $0.213 < r < 0.250$  [28]. This is extremely interesting result as it is different from other class of DBI models. The most impressive fact is that the upper bounds of  $f_{NL;1}^{equil}$  in both the limits are within the future observational bound as predicted by PLANCK [2]. The graphical behavior of  $f_{NL;1}^{equil}$  with respect to the sound speed ( $c_s$ ) and tensor to scalar ratio ( $r$ ) are plotted in the figure(1)(a) and figure(1)(b). It is evident from the figure(1) that  $f_{NL;1}^{equil}$  is not much sensitive to the tensor to scalar ratio ( $r$ ) or the sound speed ( $c_s$ ) within a specified range applicable for our model in  $DS$  and  $BDS$  limit analysis.

### B. One scalar two tensor correlation

The one scalar and two tensor interaction for D3 DBI Galileon can be represented in uniform gauge by the following third order perturbative action:

$$\begin{aligned} (S^{(4)})_{\zeta hh} = \int dt d^3x a^3 \left\{ \mathcal{F}_1 \dot{\zeta} h_{ij}^2 + \frac{\tilde{\mathcal{F}}_2}{a^2} \zeta h_{ij,k} h_{ij,k} + \tilde{\mathcal{F}}_3 \psi_{,k} \dot{h}_{ij} h_{ij,k} + \mathcal{F}_4 \dot{\zeta} \dot{h}_{ij}^2 + \frac{\tilde{\mathcal{F}}_5}{a^2} \partial^2 \zeta \dot{h}_{ij}^2 \right. \\ \left. + \tilde{\mathcal{F}}_6 \psi_{,ij} \dot{h}_{ik} \dot{h}_{jk} + \frac{\tilde{\mathcal{F}}_7}{a^2} \zeta_{,ij} \dot{h}_{ik} \dot{h}_{jk} \right\} + \int dt d^3x \mathcal{S}_{\zeta hh} \end{aligned} \quad (3.18)$$

where the dimensionful coefficients  $\mathcal{F}_i (i = 1, 2, \dots, 7)$  are defined as:

$$\begin{aligned} \tilde{\mathcal{F}}_1 &= 3Y_T \left[ 1 - \frac{HL_1 Y_T}{c_T^2} + \frac{Y_T}{3} \frac{d}{dt} \left( \frac{L_1}{c_T^2} \right) \right], \quad \tilde{\mathcal{F}}_2 = Y_s c_s^2, \quad \tilde{\mathcal{F}}_3 = -2Y_s, \\ \tilde{\mathcal{F}}_4 &= \frac{L_1}{c_T^2} \left( Y_T^2 - \hat{K}_{XX} \right) + 2\sigma \left[ \frac{Y_s}{Y_T} - 1 - \frac{HL_1 Y_T}{c_T^2} \left( 6 + \frac{\dot{Y}_s}{HY_s} \right) \right] + 2Y_T^2 \frac{d}{dt} \left( \frac{\sigma L_1}{Y_T c_T^2} \right), \\ \tilde{\mathcal{F}}_5 &= 2\sigma Y_T L_1 \left( \frac{c_s^2}{c_T^2} - 1 \right), \quad \tilde{\mathcal{F}}_6 = -\frac{4\sigma Y_s}{Y_T}, \quad \tilde{\mathcal{F}}_7 = 4\sigma Y_T L_1 \end{aligned} \quad (3.19)$$

where we use  $\sigma = \dot{\phi} X G_{5X}$ . The extra term in the action represented in equation(3.18) can be expressed as

$$\mathcal{S}_{\zeta hh} = \frac{2\sigma}{Y_s} \frac{L_1 Y_T}{c_T^2} \dot{h}_{ij}^2 \mathcal{E}^s + \frac{4L_1 Y_T}{c_T^2} \left( \frac{\zeta}{2} + \frac{\sigma}{Y_T} \dot{\zeta} \right) \dot{h}_{ij} \mathcal{E}_{ij}^T \quad (3.20)$$

with the following gauge fixing condition:

$$\mathcal{E}^s : \partial_t \left( a^3 Y_S \dot{\zeta} \right) - a Y_S c_s^2 \partial^2 \zeta = 0, \quad (3.21)$$

$$\mathcal{E}_{ij}^T : \partial_t \left( a^3 Y_T \dot{h}_{ij} \right) - a Y_T c_T^2 \partial^2 h_{ij} = 0. \quad (3.22)$$

The contribution from the last term in the above action can be gauged away by making use of the following gauge transformation equations along with the gauge fixing condition for tensor and scalar modes:

$$\begin{aligned} h_{ij} &\rightarrow h_{ij} + \frac{L_1 Y_T}{c_T^2} \left( \zeta + \frac{2\sigma}{Y_T} \dot{\zeta} \right) \dot{h}_{ij}, \\ \zeta &\rightarrow \zeta + \frac{\sigma}{8Y_S} \frac{L_1 Y_T}{c_T^2} \dot{h}_{ij}^2. \end{aligned} \quad (3.23)$$

Now following the prescription of *in-in formalism* in the interaction picture *three point one scalar two tensor correlation function* both for *DS* and *BDS* can be expressed in the following form:

$$\begin{aligned} \langle \zeta(\vec{k}_1) h_{ij}(\vec{k}_2) h_{kl}(\vec{k}_3) \rangle &= -i \sum_{q=1}^7 \int_{-\infty}^0 d\eta \, a \, \langle 0 | \left[ \zeta(\vec{k}_1) h_{ij}(\vec{k}_2) h_{kl}(\vec{k}_3), \left( [H_{int}^{(q)}(\eta)]_{ij;kl} \right)_{\zeta hh} \right] | 0 \rangle \\ &= (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \{B_{\zeta hh}\}_{ij;kl}(\vec{k}_1, \vec{k}_2, \vec{k}_3), \end{aligned} \quad (3.24)$$

where the total Hamiltonian in the interaction picture can be expressed in terms of the third order Lagrangian density as  $\left( [H_{int}(\eta)]_{ij;kl} \right)_{\zeta hh} = \sum_{q=1}^7 \left( [H_{int}^{(q)}(\eta)]_{ij;kl} \right)_{\zeta hh} = - \int d^3x \left[ (\mathcal{L}_3)_{\zeta hh} \right]_{ij;kl}$ . Moreover the *cross bispectrum*  $\{B_{\zeta hh}\}_{ij;kl}(\vec{k}_1, \vec{k}_2, \vec{k}_3)$  is defined as:

$$\{B_{\zeta hh}\}_{ij;kl}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{(2\pi)^4 \mathcal{P}_u^2}{\prod_{i=1}^3 k_i^3} (\mathcal{A}_{\zeta hh})_{ij;kl}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{6}{5} [f_{NL;2}^{local}]_{ij;kl}^u P_u^2 \quad (3.25)$$

where the symbol ;2 stands for one scalar two tensor correlation. Here  $(\mathcal{A}_{\zeta hh})_{ij;kl}(\vec{k}_1, \vec{k}_2, \vec{k}_3)$  is the *shape function* for bispectrum and the polarization indices are  $u = 1(E - mode), 2(E \otimes B - mode), 3(B - mode)$ . We adopt the following normalization depending on the polarization in which we are interested in:

$$P_u^2 = \begin{cases} P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1) & :u=1(E-mode) \\ P_\zeta(k_1)P_h(k_2) + P_\zeta(k_2)P_h(k_3) + P_\zeta(k_3)P_h(k_1) & :u=2(E \otimes B mode) \\ P_h(k_1)P_h(k_2) + P_h(k_2)P_h(k_3) + P_h(k_3)P_h(k_1) & :u=3(B-mode). \end{cases} \quad (3.26)$$

Consequently  $[f_{NL;2}^{local}]_{ij;kl}^u$  represents the non-linear parameter which carries the signature of primordial non-Gaussianities of the one scalar two tensor interaction. The explicit form of  $[f_{NL;2}^{local}]_{ij;kl}^u$  characterizing the bispectrum can be calculated as:

$$\begin{aligned} [f_{NL;2}^{local}]_{ij;kl}^u \Big|_{DS} &= \frac{10H^2 Q_u^{POL} Y_s c_s}{3 \sum_{i=1}^3 k_i^3 Y_T^2 c_T^2} \sum_{p=1}^7 \left\{ \tilde{\mathcal{F}}_p \left[ \gamma_{ij;kl}^{(p)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \right]^u \mathcal{G}^{(p)}(k_1, k_2, k_3) \right. \\ &\quad \left. + \tilde{\mathcal{F}}_p \left[ \gamma_{kl;ij}^{(p)}(\vec{k}_1, \vec{k}_3, \vec{k}_2) \right]^u \mathcal{G}^{(p)}(k_1, k_3, k_2) \right\} \end{aligned} \quad (3.27)$$

where in the *DS* limit the characteristic polarized coefficients in the tensor basis are given by:

$$\begin{aligned} [\gamma_{ij,kl}^{(1)}]^u &= \sum_{m,n} [\mathcal{N}_{ij,mn}(\vec{k}_2) \mathcal{N}_{kl,mn}(\vec{k}_3)]^u, \quad [\gamma_{ij,kl}^{(2)}]^u = \vec{k}_2 \cdot \vec{k}_3 [\gamma_{ij,kl}^{(1)}]^u, \quad [\gamma_{ij,kl}^{(3)}]^u = \frac{\vec{k}_1 \cdot \vec{k}_3}{k_1^2} [\gamma_{ij,kl}^{(1)}]^u, \\ [\gamma_{ij,kl}^{(4)}]^u &= [\gamma_{ij,kl}^{(1)}]^u, \quad [\gamma_{ij,kl}^{(5)}]^u = k_1^2 [\gamma_{ij,kl}^{(1)}]^u, \quad [\gamma_{ij,kl}^{(6)}]^u = \sum_{m,n} \hat{k}_{1m} \hat{k}_{1n} \left[ \sum_{m'} \mathcal{N}_{ij,mm'}(\vec{k}_2) \mathcal{N}_{kl,nm'}(\vec{k}_3) \right]^u, \\ [\gamma_{ij,kl}^{(7)}]^u &= k_1^2 [\gamma_{ij,kl}^{(6)}]^u \end{aligned} \quad (3.28)$$



with polarization index  $u = 1(E), 2(E \otimes B), 3(B)$  and  $\mathcal{N}_{ij;kl} = \left( \sum_{\lambda} e_{ij}^{\lambda}(\vec{k}) e_{kl}^{\dagger(\lambda)}(\vec{k}) \right)$ . In this context the momentum dependent functions can be written as:

$$\begin{aligned} \mathcal{G}^{(1)} &= \frac{1}{H^2} \frac{c_T^4 k_2^2 k_3^2 (c_s k_1 + \underline{K})}{\underline{K}^2}, \\ \mathcal{G}^{(2)} &= -\frac{1}{H^2} \frac{c_s^3 k_1^3 + 2c_s^2 c_T k_1^2 (k_2 + k_3) + 2c_s c_T^2 k_1 (k_2^2 + k_2 k_3 + k_3^2) + c_T^3 (k_2 + k_3) (k_2^2 + k_2 k_3 + k_3^2)}{\underline{K}^2}, \\ \mathcal{G}^{(3)} &= \frac{1}{H^2} \frac{c_s^2 c_T^2 k_1^2 k_2^2 (\underline{K} + c_T k_3)}{\underline{K}^2}, \quad \mathcal{G}^{(4)} = \frac{2}{H} \frac{c_s^2 c_T^4 k_1^2 k_2^2 k_3^2}{\underline{K}^3}, \quad \mathcal{G}^{(5)} = \frac{2c_T^4 k_2^2 k_3^2 (3c_s k_1 + \underline{K})}{\underline{K}^4}, \\ \mathcal{G}^{(6)} &= \mathcal{G}^{(4)}, \quad \mathcal{G}^{(7)} = \mathcal{G}^{(5)} \end{aligned} \quad (3.29)$$

where we define  $\underline{K} := c_s k_1 + c_T (k_2 + k_3)$ . In BDS limit we have

$$\begin{aligned} \left[ [f_{NL;2}^{local}]^u_{ij;kl} \right]_{BDS} &= \frac{10 \mathcal{Q}_u^{POL} \left( \frac{3}{2} - \nu_T \right)^2 \underline{K}^{4\nu_T + 2\nu_s - 9} \left[ \text{Cos} \left( \left[ \nu_s - \frac{1}{2} \right] \frac{\pi}{2} \right) \right]^{\frac{1}{3}} \left[ \text{Cos} \left( \left[ \nu_T - \frac{1}{2} \right] \frac{\pi}{2} \right) \right]^{\frac{2}{3}}}{3 \sum_{i=1}^3 k_i^3 \frac{c_s^{2\nu_s - 3} c_T^{4\nu_T - 6} (k_1)^{\nu_s} (k_2 k_3)^{\nu_T}}{2^{2\nu_s + 4\nu_T - 12} \left| \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right|^2 \left| \frac{\Gamma(\nu_T)}{\Gamma(\frac{3}{2})} \right|^4 \frac{(1 - \epsilon_V - s_V^S)^2 (1 - \epsilon_V - s_V^T)^4 V^{\frac{3}{2}}(\phi)}}{Y_S Y_T^2 c_T^4 c_s^3 \tilde{g}_1^{\frac{3}{2}} M_{PL}^3}} \\ &\times \left[ 32 \tilde{\mathcal{F}}_1 (\nabla_1)_{ij;kl}^u + 4 \tilde{\mathcal{F}}_2 (\nabla_2)_{ij;kl}^u + 2 \left( \tilde{\mathcal{F}}_3 (\nabla_3)_{ij;kl}^u \right. \right. \\ &\quad \left. \left. + \tilde{\mathcal{F}}_4 (\nabla_4)_{ij;kl}^u + \tilde{\mathcal{F}}_5 (\nabla_5)_{ij;kl}^u + \tilde{\mathcal{F}}_6 (\nabla_6)_{ij;kl}^u + \tilde{\mathcal{F}}_7 (\nabla_7)_{ij;kl}^u \right) \right] \end{aligned} \quad (3.30)$$

where in the *BDS* limit

$$\begin{aligned} (\nabla_1)_{ij;kl}^u &= \sum_{p=1}^6 \left\{ \frac{[\mathcal{J}_p(\vec{k}_1, \vec{k}_2, \vec{k}_3)]_{ij;kl}^u}{k_1^{\nu_s} (k_2 k_3)^{\nu_T}} + \frac{[\mathcal{J}_p(\vec{k}_2, \vec{k}_1, \vec{k}_3)]_{ij;kl}^u}{k_2^{\nu_s} (k_1 k_3)^{\nu_T}} + \frac{[\mathcal{J}_p(\vec{k}_3, \vec{k}_2, \vec{k}_1)]_{ij;kl}^u}{k_3^{\nu_s} (k_2 k_1)^{\nu_T}} \right\} \frac{\Gamma(7 + p - 4\nu_T - 2\nu_s)}{c_s^{2\nu_s - \frac{p}{3} - \frac{7}{3}} c_T^{4\nu_T - \frac{2p}{3} - \frac{14}{3}} \underline{K}^{7 + p - 4\nu_T - 2\nu_s}}, \\ (\nabla_2)_{ij;kl}^u &= \frac{\left[ \frac{(\vec{k}_1, \vec{k}_2)}{k_1^{\nu_s} (k_2 k_3)^{\nu_T}} + \frac{(\vec{k}_2, \vec{k}_3)}{k_2^{\nu_s} (k_1 k_3)^{\nu_T}} + \frac{(\vec{k}_3, \vec{k}_1)}{k_3^{\nu_s} (k_2 k_3)^{\nu_T}} \right]}{\left( \frac{3}{2} - \nu_T \right)^2 c_T^2} \sum_{p=1}^4 H_p \frac{\Gamma(9 + p - 4\nu_T - 2\nu_s)}{c_s^{2\nu_s - \frac{p}{3} - 3} c_T^{4\nu_T - \frac{2p}{3} - 6} \underline{K}^{9 + p - 4\nu_T - 2\nu_s}} \mathcal{N}_{ij,kl}^u, \\ (\nabla_3)_{ij;kl}^u &= \frac{\left[ (\vec{k}_1, \vec{k}_2) Y_{123} + (\vec{k}_1, \vec{k}_3) Y_{132} + (\vec{k}_2, \vec{k}_3) Y_{213} + (\vec{k}_2, \vec{k}_1) Y_{231} + (\vec{k}_3, \vec{k}_2) Y_{312} + (\vec{k}_3, \vec{k}_1) Y_{321} \right] \mathcal{N}_{ij,kl}^u}{\left( \frac{3}{2} - \nu_T \right)^2 c_T^2}, \\ (\nabla_4)_{ij;kl}^u &= \left( \frac{3}{2} - \nu_s \right) \left[ \tilde{\mathcal{J}}_{123} + \tilde{\mathcal{J}}_{132} + \tilde{\mathcal{J}}_{213} + \tilde{\mathcal{J}}_{231} + \tilde{\mathcal{J}}_{312} + \tilde{\mathcal{J}}_{321} \right] \mathcal{N}_{ij,kl}^u, \\ (\nabla_5)_{ij;kl}^u &= \left[ \tilde{\mathcal{C}}_{123} + \tilde{\mathcal{C}}_{132} + \tilde{\mathcal{C}}_{213} + \tilde{\mathcal{C}}_{231} + \tilde{\mathcal{C}}_{312} + \tilde{\mathcal{C}}_{321} \right]_{ij,kl}^u, \\ (\nabla_6)_{ij;kl}^u &= \left[ \hat{\mathcal{W}}_{123} + \hat{\mathcal{W}}_{132} + \hat{\mathcal{W}}_{213} + \hat{\mathcal{W}}_{231} + \hat{\mathcal{W}}_{312} + \hat{\mathcal{W}}_{321} \right] \mathcal{N}_{ij,kl}^u, \\ (\nabla_7)_{ij;kl}^u &= \left[ k_{1m} k_{1m'} \{ \bar{X}_{123} + \bar{X}_{132} \} + k_{2m} k_{2m'} \{ \bar{X}_{231} + \bar{X}_{213} \} + k_{3m} k_{3m'} \{ \bar{X}_{312} + \bar{X}_{321} \} \right] \mathcal{N}_{ij,kl}^u \mathcal{N}_{mn,m'n}^u. \end{aligned} \quad (3.31)$$

with

$$\begin{aligned} \left[ \mathcal{J}_1(\vec{k}_a, \vec{k}_b, \vec{k}_c) \right]_{ij;kl}^u &= \mathcal{N}_{ij,kl}^u \left( \frac{3}{2} - \nu_T \right)^2, \quad \left[ \mathcal{J}_2(\vec{k}_a, \vec{k}_b, \vec{k}_c) \right]_{ij;kl}^u = \left[ \mathcal{J}_1(\vec{k}_a, \vec{k}_b, \vec{k}_c) \right]_{ij;kl}^u \underline{K} c_s^{-\frac{1}{3}} c_T^{-\frac{2}{3}}, \\ \left[ \mathcal{J}_3(\vec{k}_a, \vec{k}_b, \vec{k}_c) \right]_{ij;kl}^u &= \mathcal{N}_{ij,kl}^u \left( \frac{3}{2} - \nu_T \right) \left[ (k_a^2 + k_b^2 + k_a k_b) + \left( \frac{3}{2} - \nu_T \right) k_a (k_b + k_c) \right] \\ \left[ \mathcal{J}_4(\vec{k}_a, \vec{k}_b, \vec{k}_c) \right]_{ij;kl}^u &= \mathcal{N}_{ij,kl}^u \left( \frac{3}{2} - \nu_T \right) \left[ k_b k_c (k_b + k_c) + k_a (k_b^2 + k_c^2 + k_b k_c) \right], \\ \left[ \mathcal{J}_5(\vec{k}_a, \vec{k}_b, \vec{k}_c) \right]_{ij;kl}^u &= \mathcal{N}_{ij,kl}^u \left[ k_b^2 k_c^2 + k_a k_b k_c (k_b + k_c) \right], \quad \left[ \mathcal{J}_6(\vec{k}_a, \vec{k}_b, \vec{k}_c) \right]_{ij;kl}^u = i \mathcal{N}_{ij,kl}^u k_a k_b^2 k_c^2, \\ &\quad (\text{includes 3 permutations of } a, b, c), \end{aligned} \quad (3.32)$$

$$\begin{aligned}
Y_{abc} &= \frac{1}{k_a^{\nu_s}(k_b k_c)^{\nu_T}} \left\{ \sum_{p=1}^4 H_p \frac{\Gamma(9+p-4\nu_T-2\nu_s)}{c_s^{2\nu_s-\frac{p}{3}-3} c_T^{4\nu_T-\frac{2p}{3}-6} \underline{K}^{9+p-4\nu_T-2\nu_s}} + \sum_{q=1}^5 \mathcal{A}_q^{abc} \frac{\Gamma(8+q-4\nu_T-2\nu_s)}{c_s^{2\nu_s-\frac{q}{3}-\frac{8}{3}} c_T^{4\nu_T-\frac{2q}{3}-\frac{16}{3}} \underline{K}^{8+q-4\nu_T-2\nu_s}} \right\} \\
&\quad (with \ a, b, c = 1, 2, 3 \quad with \ a \neq b \neq c), \\
H_1 &= H_2 = L_1, H_3 = \frac{H_1 (k_a k_b + k_b k_c + k_c k_a)}{c_s^{\frac{2}{3}} c_T^{\frac{4}{3}} \underline{K}^2}, H_4 = -\frac{k_a k_b k_c}{c_s^{-1} c_T^{-2} \underline{K}^3}, \\
\mathcal{A}_1^{abc} &= \frac{a^2 Y_s c_s^2}{t_1} \left( \frac{3}{2} - \nu_s \right)^2, \mathcal{A}_2^{abc} = c_s^{-\frac{1}{3}} c_T^{-\frac{2}{3}} \underline{K} \mathcal{A}_1^{abc}, \mathcal{A}_3^{abc} = -(k_a k_b + k_b k_c + k_c k_a + k_a^2) \mathcal{A}_1^{abc}, \\
\mathcal{A}_4^{abc} &= -\frac{a^2 Y_s c_s^2}{t_1} \left( \frac{3}{2} - \nu_s \right) \left[ k_a k_b k_c \left( \frac{3}{2} - \nu_s \right) + k_a^2 (k_b + k_c) \right], \mathcal{A}_5^{abc} = \frac{a^2 Y_s c_s^2}{t_1} k_a^2 k_b k_c,
\end{aligned} \tag{3.33}$$

$$\begin{aligned}
\tilde{\mathcal{J}}_{abc} &= \sum_{p=1}^7 \frac{a_p^{abc}}{k_a^{\nu_s}(k_b k_c)^{\nu_T}} \frac{\Gamma(6+p-4\nu_T-2\nu_s)}{c_s^{2\nu_s-\frac{p}{3}-2} c_T^{4\nu_T-\frac{2p}{3}-4} \underline{K}^{6+p-4\nu_T-2\nu_s}}, a_1^{abc} = \left( \frac{3}{2} - \nu_T \right)^2 \left( \frac{3}{2} - \nu_s \right), a_2^{abc} = a_1^{abc} c_s^{-\frac{1}{3}} c_T^{-\frac{2}{3}} \underline{K}, \\
a_3^{abc} &= \left( \frac{3}{2} - \nu_T \right) \left( \frac{3}{2} - \nu_s \right) [k_a(k_b + k_c) + k_b^2 + k_c^2 + k_b k_c] + \left( \frac{3}{2} - \nu_T \right)^2 k_a^2, \\
a_4^{abc} &= \left[ k_a^2 (k_b + k_c) \left( \frac{3}{2} - \nu_T \right) + \{k_a(k_b^2 + k_c^2 + k_b k_c) + k_b k_c (k_b + k_c)\} \left( \frac{3}{2} - \nu_T \right) \left( \frac{3}{2} - \nu_s \right) \right], \\
a_5^{abc} &= \left[ \left( \frac{3}{2} - \nu_T \right) k_a^2 (k_b^2 + k_c^2 + k_b k_c) + \left( \frac{3}{2} - \nu_s \right) k_b^2 k_c^2 + \left( \frac{3}{2} - \nu_T \right) \left( \frac{3}{2} - \nu_s \right) k_a k_b k_c (k_b + k_c) \right], \\
a_6^{abc} &= k_a k_b k_c \left[ \left( \frac{3}{2} - \nu_T \right) k_b k_c + \left( \frac{3}{2} - \nu_s \right) k_a (k_b + k_c) \right], a_7^{abc} = -k_a^2 k_b^2 k_c^2,
\end{aligned} \tag{3.34}$$

$$\begin{aligned}
[\tilde{\mathcal{C}}_{abc}]_{ij;kl}^u &= \sum_{p=1}^6 \frac{[\mathcal{J}_p(\vec{k}_a, \vec{k}_b, \vec{k}_c)]_{ij;kl}^u}{k_a^{\nu_s}(k_b k_c)^{\nu_T}} \frac{\Gamma(7+p-4\nu_T-2\nu_s)}{c_s^{2\nu_s-\frac{p}{3}-\frac{7}{3}} c_T^{4\nu_T-\frac{2p}{3}-\frac{14}{3}} \underline{K}^{7+p-4\nu_T-2\nu_s}} \quad (includes \ 6 \text{ permutations of } a, b, c), \\
\hat{\mathcal{W}}_{abc} &= \frac{1}{k_a^{\nu_s}(k_b k_c)^{\nu_T}} \left\{ k_a^2 \frac{\Gamma(8-4\nu_T-2\nu_s)}{c_s^{2\nu_s-\frac{8}{3}} c_T^{4\nu_T-\frac{16}{3}} \underline{K}^{8-4\nu_T-2\nu_s}} + \frac{a^2 Y_s c_s^2}{t_1} \sum_{p=1}^7 \frac{a_p^{abc}}{c_s^{2\nu_s-\frac{p}{3}-2} c_T^{4\nu_T-\frac{2p}{3}-4} \underline{K}^{6+p-4\nu_T-2\nu_s}} \right\}, \\
\bar{X}_{abc} &= \sum_{p=1}^7 \frac{a_p^{abc}}{k_a^{\nu_s}(k_b k_c)^{\nu_T}} \frac{\Gamma(7+p-4\nu_T-2\nu_s)}{c_s^{2\nu_s-\frac{p}{3}-\frac{7}{3}} c_T^{4\nu_T-\frac{2p}{3}-\frac{14}{3}} \underline{K}^{7+p-4\nu_T-2\nu_s}}.
\end{aligned} \tag{3.35}$$

For both *DS* and *BDS* limit the overall normalization factor for three types of polarization can be expressed as:

$$\mathcal{Q}_u^{POL} = \begin{cases} 8 & :u=1(E\text{-mode}) \\ 128 & :u=2(E \otimes B \text{ mode}) \\ 2048 & :u=3(B\text{-mode}). \end{cases} \tag{3.36}$$

Further, to make the computation simpler without losing any essential information we reduce the complete set in terms of the two-polarization (helicity) mode instead of four complicated tensor indices. For this purpose let us define a reduced physical quantity:

$$\bigoplus^\lambda (\vec{k}) = h_{ij}(\vec{k}) e_{ij}^{\dagger(\lambda)} \tag{3.37}$$

in terms of which the one scalar two tensor correlation is defined as:

$$\langle \zeta(\vec{k}_1) \bigoplus^{\lambda_2} (\vec{k}_2) \bigoplus^{\lambda_3} (\vec{k}_3) \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_{(\zeta hh)}^{(\lambda_1; \lambda_2)}(\vec{k}_1, \vec{k}_2, \vec{k}_3). \tag{3.38}$$

where the *cross reduced bispectrum* is defined as:

$$B_{(\zeta hh)}^{(\lambda_2; \lambda_3)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{(2\pi)^4 \mathcal{P}_u^2}{\prod_{i=1}^3 k_i^3} \mathcal{A}_{(\zeta hh)}^{(\lambda_2; \lambda_3)} = \frac{6}{5} [f_{NL;2}^{local}]^{u;(\lambda_2; \lambda_3)} P_u^2. \tag{3.39}$$

Here applying the basis transformation the explicit form of  $[f_{NL;2}^{local}]^{(\lambda_2;\lambda_3)}$  characterizing the crossed bispectrum can be calculated as:

$$\begin{aligned} [f_{NL;2}^{local}]^{u;(\lambda_2;\lambda_3)}]_{DS} &= \frac{10H^2 \mathcal{Q}_u^{POL}}{3 \sum_{i=1}^3 k_i^3} \frac{Y_s c_s}{Y_T^2 c_T^2} \sum_{p=1}^7 \left\{ \tilde{\mathcal{F}}_p \left[ \gamma_{\lambda_2, \lambda_3}^{(p)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \right]^u \mathcal{G}^{(p)}(k_1, k_2, k_3) \right. \\ &\quad \left. + \tilde{\mathcal{F}}_p \left[ \gamma_{\lambda_3, \lambda_2}^{(p)}(\vec{k}_1, \vec{k}_3, \vec{k}_2) \right]^u \mathcal{G}^{(p)}(k_1, k_3, k_2) \right\} \end{aligned} \quad (3.40)$$

where the DS polarized coefficients in the helicity basis  $(\left[ \gamma_{\lambda_2, \lambda_3}^{(q)}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \right]^u)$  can be evaluated as

$$\begin{aligned} \left[ \gamma_{\lambda_2, \lambda_3}^{(1)} \right]^u &= \frac{1}{16k_2^2 k_3^2} [k_1^2 - (\lambda_2 k_2 + \lambda_3 k_3)^2]^2, \quad \left[ \gamma_{\lambda_2, \lambda_3}^{(2)} \right]^u = \vec{k}_2 \cdot \vec{k}_3 \left[ \gamma_{\lambda_2, \lambda_3}^{(1)} \right]^u = \frac{k_1^2 - k_2^2 - k_3^2}{2} \left[ \gamma_{\lambda_2, \lambda_3}^{(1)} \right]^u, \\ \left[ \gamma_{\lambda_2, \lambda_3}^{(3)} \right]^u &= \frac{\vec{k}_1 \cdot \vec{k}_3}{k_1^2} \left[ \gamma_{\lambda_2, \lambda_3}^{(1)} \right]^u = -\frac{k_1^2 - k_2^2 + k_3^2}{2} \left[ \gamma_{\lambda_2, \lambda_3}^{(1)} \right]^u, \quad \left[ \gamma_{\lambda_2, \lambda_3}^{(4)} \right]^u = \left[ \gamma_{\lambda_2, \lambda_3}^{(1)} \right]^u, \quad \left[ \gamma_{\lambda_2, \lambda_3}^{(5)} \right]^u = k_1^2 \left[ \gamma_{\lambda_2, \lambda_3}^{(1)} \right]^u, \\ \left[ \gamma_{\lambda_2, \lambda_3}^{(6)} \right]^u &= \frac{K}{32k_1^2 k_2^2 k_3^2} (k_1 - k_2 - k_3)(k_1 + k_2 - k_3)(k_1 - k_2 + k_3) [k_1^2 - (\lambda_2 k_2 + \lambda_3 k_3)^2]^u, \\ \left[ \gamma_{\lambda_2, \lambda_3}^{(7)} \right]^u &= k_1^2 \left[ \gamma_{\lambda_2, \lambda_3}^{(6)} \right]^u \end{aligned} \quad (3.41)$$

and in BDS limit

$$\begin{aligned} [f_{NL;2}^{local}]^{u;(\lambda_2;\lambda_3)}]_{BDS} &= \frac{10 \mathcal{Q}_u^{POL}}{3 \sum_{i=1}^3 k_i^3} \frac{(\frac{3}{2} - \nu_T)^2 \underline{K}^{4\nu_T + 2\nu_s - 9} [Cos([\nu_s - \frac{1}{2}][\frac{\pi}{2}])]^{\frac{1}{3}} [Cos([\nu_T - \frac{1}{2}][\frac{\pi}{2}])]^{\frac{2}{3}}}{c_s^{2\nu_s - 3} c_T^{4\nu_T - 6} (k_1)^{\nu_s} (k_2 k_3)^{\nu_T}} \\ &\quad \times \left( 2^{2\nu_s + 4\nu_T - 12} \left| \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right|^2 \left| \frac{\Gamma(\nu_T)}{\Gamma(\frac{3}{2})} \right|^4 \frac{(1 - \epsilon_V - s_V^S)^2 (1 - \epsilon_V - s_V^T)^4 V^{\frac{3}{2}}(\phi)}{Y_S Y_T^2 c_T^4 c_s^3 \tilde{g}_1^{\frac{3}{2}} M_{PL}^3} \right) \\ &\quad \times \left[ 32 \tilde{\mathcal{F}}_1 (\nabla_1)^{u; \lambda_2; \lambda_3} + 4 \tilde{\mathcal{F}}_2 (\nabla_2)^{u; \lambda_2; \lambda_3} + 2 \left( \tilde{\mathcal{F}}_3 (\nabla_3)^{u; \lambda_2; \lambda_3} \right. \right. \\ &\quad \left. \left. + \tilde{\mathcal{F}}_4 (\nabla_4)^{u; \lambda_2; \lambda_3} + \tilde{\mathcal{F}}_5 (\nabla_5)^{u; \lambda_2; \lambda_3} + \tilde{\mathcal{F}}_6 (\nabla_6)^{u; \lambda_2; \lambda_3} + \tilde{\mathcal{F}}_7 (\nabla_7)^{u; \lambda_2; \lambda_3} \right) \right] \end{aligned} \quad (3.42)$$

where in the *BDS* case

$$\begin{aligned} (\nabla_1)^{u; \lambda_2; \lambda_3} &= \sum_{p=1}^6 \left\{ \frac{[\mathcal{J}_p(\vec{k}_1, \vec{k}_2, \vec{k}_3)]^{u; \lambda_2; \lambda_3}}{k_1^{\nu_s} (k_2 k_3)^{\nu_T}} + \frac{[\mathcal{J}_p(\vec{k}_2, \vec{k}_1, \vec{k}_3)]^{u; \lambda_2; \lambda_3}}{k_2^{\nu_s} (k_1 k_3)^{\nu_T}} + \frac{[\mathcal{J}_p(\vec{k}_3, \vec{k}_2, \vec{k}_1)]^{u; \lambda_2; \lambda_3}}{k_3^{\nu_s} (k_2 k_1)^{\nu_T}} \right\} \\ &\quad \times \frac{\Gamma(7 + p - 4\nu_T - 2\nu_s)}{c_s^{2\nu_s - \frac{p}{3} - \frac{7}{3}} c_T^{4\nu_T - \frac{2p}{3} - \frac{14}{3}} \underline{K}^{7 + p - 4\nu_T - 2\nu_s}}, \\ (\nabla_2)^{u; \lambda_2; \lambda_3} &= \frac{2 \left[ \frac{(\vec{k}_1 \cdot \vec{k}_2)}{k_1^{\nu_s} (k_2 k_3)^{\nu_T}} + \frac{(\vec{k}_2 \cdot \vec{k}_3)}{k_2^{\nu_s} (k_1 k_3)^{\nu_T}} + \frac{(\vec{k}_3 \cdot \vec{k}_1)}{k_3^{\nu_s} (k_1 k_2)^{\nu_T}} \right]^{u; \lambda_2; \lambda_3}}{(\frac{3}{2} - \nu_T)^2 c_T^2} \sum_{p=1}^4 H_p \frac{\Gamma(9 + p - 4\nu_T - 2\nu_s)}{c_s^{2\nu_s - \frac{p}{3} - 3} c_T^{4\nu_T - \frac{2p}{3} - 6} \underline{K}^{9 + p - 4\nu_T - 2\nu_s}}, \\ (\nabla_3)^{u; \lambda_2; \lambda_3} &= \frac{2 \left[ (\vec{k}_1 \cdot \vec{k}_2) Y_{123} + (\vec{k}_1 \cdot \vec{k}_3) Y_{132} + (\vec{k}_2 \cdot \vec{k}_3) Y_{213} + (\vec{k}_2 \cdot \vec{k}_1) Y_{231} + (\vec{k}_3 \cdot \vec{k}_2) Y_{312} + (\vec{k}_3 \cdot \vec{k}_1) Y_{321} \right]^{u; \lambda_2; \lambda_3}}{(\frac{3}{2} - \nu_T)^2 c_T^2}, \\ (\nabla_4)^{u; \lambda_2; \lambda_3} &= 2 \left( \frac{3}{2} - \nu_s \right) \left[ \tilde{\mathcal{J}}_{123} + \tilde{\mathcal{J}}_{132} + \tilde{\mathcal{J}}_{213} + \tilde{\mathcal{J}}_{231} + \tilde{\mathcal{J}}_{312} + \tilde{\mathcal{J}}_{321} \right] \delta^{\lambda_2 \lambda_3}, \\ (\nabla_5)^{u; \lambda_2; \lambda_3} &= \left[ \tilde{\mathcal{C}}_{123} + \tilde{\mathcal{C}}_{132} + \tilde{\mathcal{C}}_{213} + \tilde{\mathcal{C}}_{231} + \tilde{\mathcal{C}}_{312} + \tilde{\mathcal{C}}_{321} \right]^{u; \lambda_2; \lambda_3}, \\ (\nabla_6)^{u; \lambda_2; \lambda_3} &= 2 \left[ \hat{\mathcal{W}}_{123} + \hat{\mathcal{W}}_{132} + \hat{\mathcal{W}}_{213} + \hat{\mathcal{W}}_{231} + \hat{\mathcal{W}}_{312} + \hat{\mathcal{W}}_{321} \right] \delta^{\lambda_2 \lambda_3}, \\ (\nabla_7)^{u; \lambda_2; \lambda_3} &= \left[ Z_1^{u; \lambda_2; \lambda_3} \{ \bar{X}_{123} + \bar{X}_{132} \} + Z_2^{u; \lambda_2; \lambda_3} \{ \bar{X}_{231} + \bar{X}_{213} \} + Z_3^{u; \lambda_2; \lambda_3} \{ \bar{X}_{312} + \bar{X}_{321} \} \right]. \end{aligned} \quad (3.43)$$

with

$$\begin{aligned}
\left[\mathcal{J}_1(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^{u;\lambda_2;\lambda_3} &= 2\left(\frac{3}{2} - \nu_T\right)^2, \left[\mathcal{J}_2(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^{u;\lambda_2;\lambda_3} = \left[\mathcal{J}_1(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^{u;\lambda_2;\lambda_3} c_s^{-\frac{1}{3}} c_T^{-\frac{2}{3}} \underline{K}, \\
\left[\mathcal{J}_3(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^{u;\lambda_2;\lambda_3} &= 2\lambda_2\lambda_3\left(\frac{3}{2} - \nu_T\right) \left[(k_a^2 + k_b^2 + k_a k_b) + \left(\frac{3}{2} - \nu_T\right) k_a(k_b + k_c)\right] \\
\left[\mathcal{J}_4(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^{u;\lambda_2;\lambda_3} &= 2\left(\frac{3}{2} - \nu_T\right) \left[\lambda_2^3 k_b k_c (k_b + k_c) + \lambda_3^3 k_a (k_b^2 + k_c^2 + k_b k_c)\right], \\
\left[\mathcal{J}_5(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^{u;\lambda_2;\lambda_3} &= 2\lambda_2^2\lambda_3^2 \left[k_b^2 k_c^2 + k_a k_b k_c (k_b + k_c)\right], \left[\mathcal{J}_6(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^{u;\lambda_2;\lambda_3} = 2i\lambda_3^2\lambda_2^2 k_a k_b^2 k_c^2, \\
&\quad (\text{includes 3 permutations of } a, b, c),
\end{aligned} \tag{3.44}$$

and

$$\begin{aligned}
\left[\tilde{\mathcal{C}}_{abc}\right]^{u;\lambda_2;\lambda_3} &= \sum_{p=1}^6 \frac{\left[\mathcal{J}_p(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^{u;\lambda_2;\lambda_3}}{k_a^{\nu_s}(k_b k_c)^{\nu_T}} \frac{\Gamma(7+p-4\nu_T-2\nu_s)}{c_s^{2\nu_s-\frac{p}{3}-\frac{7}{3}} c_T^{4\nu_T-\frac{2p}{3}-\frac{14}{3}} \underline{K}^{7+p-4\nu_T-2\nu_s}} \quad (\text{includes 6 permutations of } a, b, c), \\
Z_a^{u;\lambda_2;\lambda_3} &= \frac{c_s^{-\frac{1}{3}} c_T^{-\frac{2}{3}} \underline{K}}{32k_a^2 k_b^2 k_c^2} (k_a - k_b - k_c)(k_a + k_b - k_c)(k_a - k_b + k_c) \left[k_a^2 - (\lambda_2 k_b + \lambda_3 k_c)^2\right]^u.
\end{aligned} \tag{3.45}$$

In the equilateral limit we have

$$\begin{aligned}
\left[[f_{NL;2}^{local}]^{\mathbf{u};(\lambda_2;\lambda_3)}\right]_{DS} &= \frac{10H^2 \mathcal{Q}_u^{POL}}{9k^3} \frac{Y_s c_s}{Y_T^2 c_T^2} \sum_{p=1}^7 \left\{ \tilde{\mathcal{F}}_p \left[ \gamma_{\lambda_2, \lambda_3}^{(p)}(\vec{k}, \vec{k}, \vec{k}) \right]^u \mathcal{G}^{(p)}(k, k, k) \right. \\
&\quad \left. + \tilde{\mathcal{F}}_p \left[ \gamma_{\lambda_3, \lambda_2}^{(p)}(\vec{k}, \vec{k}, \vec{k}) \right]^u \mathcal{G}^{(p)}(k, k, k) \right\}
\end{aligned} \tag{3.46}$$

$$\begin{aligned}
\left[[f_{NL;2}^{local}]^{\mathbf{u};(\lambda_2;\lambda_3)}\right]_{BDS} &= \frac{10\mathcal{Q}_u^{POL}}{9k^3} \frac{\left(\frac{3}{2} - \nu_T\right)^2 ((c_s + 2c_T)k)^{4\nu_T+2\nu_s-9} \left[\text{Cos}\left(\left[\nu_s - \frac{1}{2}\right] \frac{\pi}{2}\right)\right]^{\frac{1}{3}} \left[\text{Cos}\left(\left[\nu_T - \frac{1}{2}\right] \frac{\pi}{2}\right)\right]^{\frac{2}{3}}}{c_s^{2\nu_s-3} c_T^{4\nu_T-6} k^{\nu_s+2\nu_T}} \\
&\quad \times \left( 2^{2\nu_s+4\nu_T-12} \left| \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right|^2 \left| \frac{\Gamma(\nu_T)}{\Gamma(\frac{3}{2})} \right|^4 \frac{(1 - \epsilon_V - s_V^S)^2 (1 - \epsilon_V - s_V^T)^4 V^{\frac{3}{2}}(\phi)}{Y_S Y_T^2 c_T^4 c_s^3 \tilde{g}_1^{\frac{3}{2}} M_{PL}^3} \right) \\
&\quad \left[ 32\tilde{\mathcal{F}}_1 (\nabla_1)^{u;\lambda_2;\lambda_3} + 4\tilde{\mathcal{F}}_2 (\nabla_2)^{u;\lambda_2;\lambda_3} + 2\left(\tilde{\mathcal{F}}_3 (\nabla_3)^{u;\lambda_2;\lambda_3}\right. \right. \\
&\quad \left. \left. + \tilde{\mathcal{F}}_4 (\nabla_4)^{u;\lambda_2;\lambda_3} + \tilde{\mathcal{F}}_5 (\nabla_5)^{u;\lambda_2;\lambda_3} + \tilde{\mathcal{F}}_6 (\nabla_6)^{u;\lambda_2;\lambda_3} + \tilde{\mathcal{F}}_7 (\nabla_7)^{u;\lambda_2;\lambda_3} \right) \right]
\end{aligned} \tag{3.47}$$

where each coefficients and functions appearing in equation((3.41)-(3.45)) are evaluated in equilateral limit.

### C. Two scalar one tensor correlation

The interactions involving one tensor and two scalars are given by the following third order perturbative action:

$$\begin{aligned}
\left(S^{(4)}\right)_{\zeta\zeta h} &= \int dt d^3x a^3 \left\{ \frac{\mathcal{Y}_1}{a^2} h_{ij} \zeta_{,i} \zeta_{,j} + \frac{\mathcal{Y}_2}{a^2} \dot{h}_{ij} \zeta_{,i} \zeta_{,j} + \mathcal{Y}_3 \dot{h}_{ij} \zeta_{,i} \psi_{,j} + \frac{\mathcal{Y}_4}{a^2} \partial^2 h_{ij} \zeta_{,i} \psi_{,j} + \frac{\mathcal{Y}_5}{a^4} \partial^2 h_{ij} \zeta_{,i} \zeta_{,j} \right. \\
&\quad \left. + \mathcal{Y}_6 \partial^2 h_{ij} \psi_{,i} \psi_{,j} \right\} + \int dt d^3x \mathcal{S}_{\zeta\zeta h}
\end{aligned} \tag{3.48}$$

where the last term of the equation(3.48) can be expressed as:

$$\begin{aligned}
\mathcal{S}_{\zeta\zeta h} &= \left[ \frac{L_1 \hat{K}_{XX}}{2} \zeta_{,j} h_{ij} + \frac{\sigma}{Y_T} \zeta_{,j} \dot{h}_{ij} + \frac{L_1 \sigma}{a^2} \zeta_{,j} \partial^2 h_{ij} - \frac{\sigma Y_s}{Y_T^2} \psi_{,j} \partial^2 h_{ij} \right] \partial^{-2} \partial_i \mathcal{E}^s \\
&\quad + \left[ \frac{L_1 Y_s}{Y_T} \left( \frac{\hat{K}_{XX}}{2} + \frac{\sigma}{L_1 Y_T} \right) \zeta_{,i} \psi_{,j} - \frac{Y_T L_1^2}{a^2} \left( \frac{\hat{K}_{XX}}{4} + \frac{\sigma}{L_1 Y_T} \right) \zeta_{,i} \zeta_{,j} \right] \mathcal{E}_{ij}^h,
\end{aligned} \tag{3.49}$$

and the dimensionful coefficients  $\mathcal{F}_i (i = 1, 2, \dots, 7)$  are defined as:

$$\begin{aligned}
\mathcal{Y}_1 &= Y_s c_s^2, \\
\mathcal{Y}_2 &= \frac{L_1 \hat{K}_{XX}}{4} (Y_s c_s^2 - Y_T c_T^2) + L_1 Y_T^2 \left[ -\frac{1}{2} + \frac{H L_1 \hat{K}_{XX}}{4} \left( 3 + \frac{\dot{Y}_T}{H Y_T} \right) - \frac{1}{4} \frac{d}{dt} (L_1 \hat{K}_{XX}) \right] \\
&\quad + \frac{\sigma Y_s c_s^2}{Y_T} + 2 H L_1 Y_T \sigma - Y_T \frac{d}{dt} (L_1 \sigma), \\
\mathcal{Y}_3 &= Y_s \left[ \frac{3}{2} + \frac{d}{dt} \left( \frac{\hat{K}_{XX} L_1}{2} + \frac{\sigma}{Y_T} \right) - \left( 3H + \frac{\dot{Y}_T}{Y_T} \right) \left( \frac{\hat{K}_{XX} L_1}{2} + \frac{\sigma}{Y_T} \right) \right], \\
\mathcal{Y}_4 &= Y_s \left[ -\frac{(Y_T - \hat{K}_{XX} c_T^2) L_1}{2} - 2H \sigma L_1 + \frac{d}{dt} (L_1 \sigma) + \frac{\sigma}{Y_T^2} (Y_T c_T^2 - Y_s c_s^2) \right], \\
\mathcal{Y}_5 &= \frac{Y_T^2 L_1}{2} \left[ \frac{(Y_T - \hat{K}_{XX} c_T^2)}{2} + 2H L_1 \sigma - \frac{d}{dt} (\sigma L_1) - \frac{\sigma}{Y_T^2} (3Y_T c_T^2 - Y_s c_s^2) \right], \\
\mathcal{Y}_6 &= \frac{Y_s^2}{4Y_T} \left[ 1 + \frac{6H\sigma}{Y_T} - 2Y_T \frac{d}{dt} \left( \frac{\sigma}{Y_T^2} \right) \right],
\end{aligned} \tag{3.50}$$

This last term of the perturbative action can be gauged away by making use of the following gauge transformations for the tensor and scalar modes:

$$h_{ij} \rightarrow h_{ij} + 4 \left[ \frac{L_1 Y_s}{Y_T} \left( \frac{\hat{K}_{XX}}{2} + \frac{\sigma}{L_1 Y_T} \right) \zeta_{,i} \psi_{,j} - \frac{Y_T L_1^2}{a^2} \left( \frac{\hat{K}_{XX}}{4} + \frac{\sigma}{L_1 Y_T} \right) \zeta_{,i} \zeta_{,j} \right], \tag{3.51}$$

$$\zeta \rightarrow \zeta - \frac{1}{2} \partial^{-2} \partial_i \left[ \frac{L_1 \hat{K}_{XX}}{2} \zeta_{,j} h_{ij} + \frac{\sigma}{Y_T} \zeta_{,j} \dot{h}_{ij} + \frac{L_1 \sigma}{a^2} \zeta_{,j} \partial^2 h_{ij} - \frac{\sigma Y_s}{Y_T^2} \psi_{,j} \partial^2 h_{ij} \right], \tag{3.52}$$

Following the prescription of *in-in formalism* in the interacting picture *three point two scalar one tensor correlation function* both for *DS* and *BDS* can be expressed in the following form:

$$\begin{aligned}
\langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) h_{kl}(\vec{k}_3) \rangle &= -i \sum_{q=1}^7 \int_{-\infty}^0 d\eta \, a \, \langle 0 | \left[ \zeta(\vec{k}_1) \zeta(\vec{k}_2) h_{kl}(\vec{k}_3), \left( [H_{int}^{(q)}(\eta)]_{kl} \right)_{\zeta \zeta h} \right] | 0 \rangle \\
&= (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \{ B_{\zeta \zeta h} \}_{kl}(\vec{k}_1, \vec{k}_2, \vec{k}_3),
\end{aligned} \tag{3.53}$$

where the total Hamiltonian in the interaction picture can be expressed in terms of the third order Lagrangian density as  $([H_{int}(\eta)]_{kl})_{\zeta \zeta h} = \sum_{q=1}^7 \left( [H_{int}^{(q)}(\eta)]_{kl} \right)_{\zeta \zeta h} = - \int d^3x \left[ (\mathcal{L}_3)_{\zeta \zeta h} \right]_{kl}$ . Here the cross bispectrum  $\{B_{\zeta \zeta h}\}_{kl}$  is defined as:

$$\{B_{\zeta \zeta h}\}_{kl} = \frac{(2\pi)^4 \mathcal{P}_u^2}{\prod_{i=1}^3 k_i^3} (\mathcal{A}_{\zeta \zeta h})_{kl} = \frac{6}{5} [f_{NL;3}^{local}]_{kl}^u P_u^2, \tag{3.54}$$

where  $(\mathcal{A}_{\zeta \zeta h})_{kl}$  is the two scalar one tensor correlation shape function and the symbol ;3 represents two scalar one tensor correlation. Consequently the non-linear parameter  $[f_{NL;3}^{local}]_{kl}^u$  can be expressed as:

$$[f_{NL;3}^{local}]_{kl}^u = \frac{10 H^2 L_u^{POL}}{3 \sum_{i=1}^3 k_i^3} \frac{1}{Y_T c_T^3} \sum_{q=1}^6 \mathcal{Y}_q \left\{ [\gamma_{kl}^{(q)}]^u(\vec{k}_1, \vec{k}_2, \vec{k}_3) \mathcal{K}^{(q)}(k_1, k_2, k_3) + [\gamma_{kl}^{(q)}]^u(\vec{k}_2, \vec{k}_1, \vec{k}_3) \mathcal{K}^{(q)}(k_2, k_1, k_3) \right\} \tag{3.55}$$

where the polarization coefficients in *DS* limit are given by:

$$\begin{aligned}
[\gamma_{kl}^{(1)}]^u &= k_{1k} k_{2l} \mathcal{N}_{ij,kl}(\vec{k}_3), & [\gamma_{kl}^{(2)}]^u &= [\gamma_{kl}^{(1)}]^u, & [\gamma_{kl}^{(3)}]^u &= \frac{1}{k_2^2} [\gamma_{kl}^{(1)}]^u, & [\gamma_{kl}^{(4)}]^u &= \frac{k_3^2}{k_2^2} [\gamma_{kl}^{(1)}]^u, \\
[\gamma_{kl}^{(5)}]^u &= k_3^2 [\gamma_{kl}^{(1)}]^u, & [\gamma_{kl}^{(6)}]^u &= \frac{k_3^2}{k_1^2 k_2^2} [\gamma_{kl}^{(1)}]^u,
\end{aligned} \tag{3.56}$$

and the momentum dependent functions are given by:

$$\begin{aligned}
\mathcal{K}^{(1)} &= -\frac{1}{H^2} \frac{c_s^3(k_1+k_2)(k_1^2+k_1k_2+k_2^2) + 2c_s^2c_T(k_1^2+k_1k_2+k_2^2)k_3 + 2c_sc_T^2(k_1+k_2)k_3^2 + c_T^3k_3^3}{\underline{\underline{K}}^2}, \\
\mathcal{K}^{(2)} &= \frac{1}{H} \frac{c_T^2k_3^2[2c_s^2(k_1^2+3k_1k_2+k_2^2) + 3c_sc_T(k_1+k_2)k_3 + c_T^2k_3^2]}{\underline{\underline{K}}^3}, \\
\mathcal{K}^{(3)} &= \frac{1}{H^2} \frac{c_s^2c_T^2k_2^2k_3^2(c_sk_1+\underline{\underline{K}})}{\underline{\underline{K}}^2}, \\
\mathcal{K}^{(4)} &= \frac{1}{H} \frac{c_s^2k_2^2[c_s^2(k_1+k_2)(2k_1+k_2) + 3c_sc_T(2k_1+k_2)k_3 + 2c_T^2k_3^2]}{\underline{\underline{K}}^3}, \\
\mathcal{K}^{(5)} &= \frac{2}{\underline{\underline{K}}^4} [c_s^3(k_1+k_2)(k_1^2+3k_1k_2+k_2^2) + 4c_s^2c_T(k_1^2+3k_1k_2+k_2^2)k_3 + 4c_sc_T^2(k_1+k_2)k_3^2 + c_T^3k_3^3], \\
\mathcal{K}^{(6)} &= \frac{1}{H^2} \frac{c_s^4k_1^2k_2^2(\underline{\underline{K}}+c_Tk_3)}{\underline{\underline{K}}^2}, \tag{3.57}
\end{aligned}$$

with  $\underline{\underline{K}} := c_s(k_1+k_2) + c_Tk_3$ .

In the *BDS* limit

$$\begin{aligned}
\left[ [f_{NL;3}^{local}]^u_{kl} \right]_{BDS} &= \frac{10L_u^{POL} \mathcal{N}_{ij;kl} \underline{\underline{K}}^{4\nu_s+2\nu_T-9} [Cos([\nu_s-\frac{1}{2}]\frac{\pi}{2})]^{\frac{2}{3}} [Cos([\nu_T-\frac{1}{2}]\frac{\pi}{2})]^{\frac{1}{3}}}{3 \sum_{i=1}^3 k_i^3 \frac{c_s^{4\nu_s-6} c_T^{2\nu_T-3} (k_1k_2)^{\nu_s} k_3^{\nu_T}}{Y_S^2 Y_T c_s^6 c_T^3 \tilde{g}_1^{\frac{3}{2}} M_{PL}^3}} \\
&\quad \times \left( 2^{4\nu_s+2\nu_T-12} \left| \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right|^4 \left| \frac{\Gamma(\nu_T)}{\Gamma(\frac{3}{2})} \right|^2 \frac{(1-\epsilon_V - s_V^S)^4 (1-\epsilon_V - s_V^T)^2 V^{\frac{3}{2}}(\phi)}{Y_S^2 Y_T c_s^6 c_T^3 \tilde{g}_1^{\frac{3}{2}} M_{PL}^3} \right) \left( \sum_{v=1}^6 \mathcal{Y}_v(\hat{\nabla}_v)_{ij} \right) \tag{3.58}
\end{aligned}$$

where in *BDS* limit we get:

$$\begin{aligned}
(\hat{\nabla}_1)_{ij} &= \left[ \frac{(k_{2i}k_{3j} + k_{3i}k_{2j})}{k_1^{\nu_T}(k_2k_3)^{\nu_s}} + \frac{(k_{1i}k_{3j} + k_{3i}k_{1j})}{k_2^{\nu_T}(k_1k_3)^{\nu_s}} + \frac{(k_{1i}k_{2j} + k_{2i}k_{1j})}{k_3^{\nu_T}(k_1k_2)^{\nu_s}} \right] \tilde{O}, \\
(\hat{\nabla}_2)_{ij} &= c_s \left( \frac{3}{2} - \nu_T \right) \left[ \frac{(k_{2i}k_{3j}P_{123} + k_{3i}k_{2j}P_{132})}{k_1^{\nu_T}(k_2k_3)^{\nu_s}} + \frac{(k_{1i}k_{3j}P_{213} + k_{3i}k_{1j}P_{231})}{k_2^{\nu_T}(k_1k_3)^{\nu_s}} + \frac{(k_{1i}k_{2j}P_{312} + k_{2i}k_{1j}P_{321})}{k_3^{\nu_T}(k_1k_2)^{\nu_s}} \right], \\
(\hat{\nabla}_3)_{ij} &= c_s \left[ \frac{(k_{2i}k_{3j}R_{123} + k_{3i}k_{2j}R_{132})}{k_1^{\nu_T}(k_2k_3)^{\nu_s}} + \frac{(k_{1i}k_{3j}R_{213} + k_{3i}k_{1j}R_{231})}{k_2^{\nu_T}(k_1k_3)^{\nu_s}} + \frac{(k_{1i}k_{2j}R_{312} + k_{2i}k_{1j}R_{321})}{k_3^{\nu_T}(k_1k_2)^{\nu_s}} \right], \\
(\hat{\nabla}_4)_{ij} &= \left[ k_1^2 \frac{(k_{2i}k_{3j}\tilde{R}_{123} + k_{3i}k_{2j}\tilde{R}_{132})}{k_1^{\nu_T}(k_2k_3)^{\nu_s}} + k_2^2 \frac{(k_{1i}k_{3j}\tilde{R}_{213} + k_{3i}k_{1j}\tilde{R}_{231})}{k_2^{\nu_T}(k_1k_3)^{\nu_s}} + k_3^2 \frac{(k_{1i}k_{2j}\tilde{R}_{312} + k_{2i}k_{1j}\tilde{R}_{321})}{k_3^{\nu_T}(k_1k_2)^{\nu_s}} \right], \\
(\hat{\nabla}_5)_{ij} &= \left[ k_1^2 \frac{(k_{2i}k_{3j} + k_{3i}k_{2j})}{k_1^{\nu_T}(k_2k_3)^{\nu_s}} + k_2^2 \frac{(k_{1i}k_{3j} + k_{3i}k_{1j})}{k_2^{\nu_T}(k_1k_3)^{\nu_s}} + k_3^2 \frac{(k_{1i}k_{2j} + k_{2i}k_{1j})}{k_3^{\nu_T}(k_1k_2)^{\nu_s}} \right] \tilde{O}, \\
(\hat{\nabla}_6)_{ij} &= \left[ k_1^2 \frac{(k_{2i}k_{3j}\tilde{L}_{123} + k_{3i}k_{2j}\tilde{L}_{132})}{k_1^{\nu_T}(k_2k_3)^{\nu_s}} + k_2^2 \frac{(k_{1i}k_{3j}\tilde{L}_{213} + k_{3i}k_{1j}\tilde{L}_{231})}{k_2^{\nu_T}(k_1k_3)^{\nu_s}} + k_3^2 \frac{(k_{1i}k_{2j}\tilde{L}_{312} + k_{2i}k_{1j}\tilde{L}_{321})}{k_3^{\nu_T}(k_1k_2)^{\nu_s}} \right] \tag{3.59}
\end{aligned}$$

with

$$\begin{aligned}
\tilde{O} &= \left\{ \sum_{p=1}^4 O_p \frac{\Gamma(9+p-4\nu_s-2\nu_T)}{c_s^{4\nu_s-\frac{2p}{3}-6} c_T^{2\nu_T-\frac{p}{3}-3} \underline{\underline{K}}^{9+p-4\nu_s-2\nu_T}} \right\}, \\
O_1 &= 1, O_2 = ic_s^{-\frac{2}{3}} c_T^{-\frac{1}{3}} \underline{\underline{K}}, O_3 = -(k_ak_b + k_bk_c + k_ck_a), O_4 = -ik_ak_bk_c, \tag{3.60}
\end{aligned}$$

$$\begin{aligned}
P_{abc} &= \sum_{p=1}^5 m_p^{abc} \frac{\Gamma(8+p-4\nu_T-2\nu_s)}{c_s^{4\nu_s-\frac{2p}{3}-\frac{16}{3}} c_T^{2\nu_T-\frac{p}{3}-\frac{8}{3}} \underline{\underline{K}}^{8+p-4\nu_s-2\nu_T}}, \\
m_1^{abc} &= \left(\frac{3}{2}-\nu_T\right), m_2^{abc} = c_s^{-\frac{2}{3}} c_T^{-\frac{1}{3}} \underline{\underline{K}} m_1^{abc}, m_3^{abc} = \left[\left(\frac{3}{2}-\nu_T\right)(k_a k_b + k_b k_c + k_c k_a) + k_a^2\right], \\
m_4^{abc} &= \left[\left(\frac{3}{2}-\nu_T\right)k_a k_b k_c + k_a^2(k_b + k_c)\right], m_5^{abc} = k_a^2 k_b k_c,
\end{aligned} \tag{3.61}$$

$$\begin{aligned}
R_{abc} &= L_1 \left(\frac{3}{2}-\nu_T\right) \sum_{p=1}^5 \bar{\mathcal{A}}_p^{abc} \frac{\Gamma(8+p-2\nu_T-4\nu_s)}{c_s^{4\nu_s-\frac{2p}{3}-\frac{16}{3}} c_T^{2\nu_T-\frac{p}{3}-\frac{8}{3}} \underline{\underline{K}}^{8+p-2\nu_T-4\nu_s}} + \left\{ \frac{a^2 Y_s \left(\frac{3}{2}-\nu_T\right) \left(\frac{3}{2}-\nu_s\right)}{t_1 k_c^2} \right. \\
&\quad \left. \times \sum_{q=1}^6 \left[\hat{\mathcal{J}}_q(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^u \frac{\Gamma(7+p-2\nu_T-4\nu_s)}{c_s^{4\nu_s-\frac{2p}{3}-\frac{14}{3}} c_T^{2\nu_T-\frac{p}{3}-\frac{7}{3}} \underline{\underline{K}}^{7+p-2\nu_T-4\nu_s}} \right\}, \\
\bar{\mathcal{A}}_1^{abc} &= \left(\frac{3}{2}-\nu_T\right), \bar{\mathcal{A}}_2^{abc} = i c_s^{-\frac{2}{3}} c_T^{-\frac{1}{3}} \underline{\underline{K}} \bar{\mathcal{A}}_1^{abc}, \bar{\mathcal{A}}_3^{abc} = -\left(\frac{3}{2}-\nu_T\right) [k_a k_b + k_b k_c + k_c k_a + k_a^2], \\
\bar{\mathcal{A}}_4^{abc} &= \left[k_a k_b k_c \left(\frac{3}{2}-\nu_T\right) + k_a^2(k_b + k_c)\right], \bar{\mathcal{A}}_5^{abc} = k_a^2 k_b k_c, \\
\left[\hat{\mathcal{J}}_1(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^u &= \left(\frac{3}{2}-\nu_s\right) \left(\frac{3}{2}-\nu_T\right), \left[\hat{\mathcal{J}}_2(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^u = i c_s^{-\frac{2}{3}} c_T^{-\frac{1}{3}} \underline{\underline{K}} \left[\hat{\mathcal{J}}_1(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^u, \\
\left[\hat{\mathcal{J}}_3(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^u &= -\left[\left(\frac{3}{2}-\nu_s\right)k_a^2 + \left(\frac{3}{2}-\nu_T\right)k_b^2 + \left(\frac{3}{2}-\nu_s\right)\left(\frac{3}{2}-\nu_T\right)\{k_a k_b + k_b k_c + k_c k_a\}\right], \\
\left[\hat{\mathcal{J}}_4(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^u &= -i \left[\left(\frac{3}{2}-\nu_s\right)k_a^2 k_c + \left(\frac{3}{2}-\nu_T\right)k_c^2 k_a + \left(\frac{3}{2}-\nu_s\right)k_a^2 k_b \right. \\
&\quad \left. + \left(\frac{3}{2}-\nu_T\right)k_c^2 k_b + k_a k_b k_c \left[\hat{\mathcal{J}}_1(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^u \right], \\
\left[\hat{\mathcal{J}}_5(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^u &= \left[k_a^2 k_c^2 + \left(\frac{3}{2}-\nu_s\right)k_a^2 k_b k_c + \left(\frac{3}{2}-\nu_T\right)k_a k_b k_c^2\right], \left[\hat{\mathcal{J}}_6(\vec{k}_a, \vec{k}_b, \vec{k}_c)\right]^u = i k_a^2 k_b k_c^2
\end{aligned} \tag{3.62}$$

and

$$\begin{aligned}
\tilde{R}_{abc} &= k_a^2 L_1 \tilde{O} + \frac{a^2 Y_s}{t_1} \frac{k_a^2}{k_b^2} \left(\frac{3}{2}-\nu_s\right) P_{abc}, \\
L_{abc} &= L_1^2 \tilde{O} - \frac{L_1 a^2 Y_s \left(\frac{3}{2}-\nu_s\right)}{t_1} \sum_{p=1}^5 n_p^{abc} \frac{\Gamma(8+p-4\nu_s-2\nu_T)}{c_s^{4\nu_s-\frac{2p}{3}-\frac{16}{3}} c_T^{2\nu_T-\frac{p}{3}-\frac{8}{3}} \underline{\underline{K}}^{8+p-4\nu_s-2\nu_T}} \\
&\quad + \frac{a^4 Y_s^2 \left(\frac{3}{2}-\nu_s\right)^2}{t_1^2 k_b^2 k_c^2} \sum_{r=1}^6 d_r^{abc} \frac{\Gamma(7+p-4\nu_s-2\nu_T)}{c_s^{4\nu_s-\frac{2p}{3}-\frac{14}{3}} c_T^{2\nu_T-\frac{p}{3}-\frac{7}{3}} \underline{\underline{K}}^{7+p-4\nu_s-2\nu_T}}, \\
n_1^{abc} &= \left(\frac{3}{2}-\nu_s\right) \left(\frac{1}{k_a^2} + \frac{1}{k_b^2}\right), n_2^{abc} = i c_s^{-\frac{2}{3}} c_T^{-\frac{1}{3}} \underline{\underline{K}} n_1^{abc}, \\
n_3^{abc} &= -\left[2 + \left(\frac{3}{2}-\nu_s\right) \left(\frac{k_c}{k_b} + \frac{k_b}{k_c}\right) + \left(\frac{3}{2}-\nu_s\right) k_a^2 (k_b + k_c) \left(\frac{1}{k_a^2} + \frac{1}{k_b^2}\right)\right], \\
n_4^{abc} &= -i \left\{ (k_c + k_b) + k_a \left[2 + \left(\frac{3}{2}-\nu_s\right) \left(\frac{k_c}{k_b} + \frac{k_b}{k_c}\right)\right] \right\}, n_5^{abc} = k_a (k_c + k_b), \\
d_1^{abc} &= \left(\frac{3}{2}-\nu_s\right)^2, d_2^{abc} = i c_s^{-\frac{2}{3}} c_T^{-\frac{1}{3}} \underline{\underline{K}} d_1^{abc}, d_3^{abc} = -(k_b^2 + k_c^2 + k_b k_c + k_a k_b + k_a k_c), d_6^{abc} = i k_a k_b^2 k_c^2 \\
d_4^{abc} &= -i \left[k_b^2 k_c^4 + \left(\frac{3}{2}-\nu_s\right) \{k_b k_c^2 + k_a (k_b^2 + k_c^2 + k_b k_c)\}\right], d_5^{abc} = \left[k_b^2 k_c^2 + k_a \left(k_b^2 k_c + k_b k_c^2 \left(\frac{3}{2}-\nu_s\right)\right)\right].
\end{aligned} \tag{3.63}$$

For both *DS* and *BDS* limit the overall normalization factor for three types of polarization can be expressed as:

$$\mathcal{L}_u^{POL} = \begin{cases} 1 & :u=1(E\text{-mode}) \\ 16 & :u=2(E\otimes B \text{ mode}) \\ 256 & :u=3(B\text{-mode}). \end{cases} \tag{3.64}$$

As mentioned in the previous sub-section, performing basis transformation cross bispectrum for two scalars and one tensor can be expressed as:

$$\langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \bigoplus^\lambda (\vec{k}_3) \rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_\lambda^{(\zeta\zeta h)}(\vec{k}_1, \vec{k}_2, \vec{k}_3). \quad (3.65)$$

where we have used the following parameterization:

$$B_\lambda^{(\zeta\zeta h)} = \frac{(2\pi)^4 \mathcal{P}_u^2}{\prod_{i=1}^3 k_i^3} \mathcal{A}_{(\zeta\zeta h)}^\lambda = \frac{6}{5} [f_{NL;3}^{local}]^{u;\lambda} P_u^2. \quad (3.66)$$

In this context the polarized non-Gaussian parameter for two scalar and one tensor mode  $[f_{NL;3}^{local}]^{u;\lambda}$  can be rewritten as:

$$[f_{NL;3}^{local}]_\lambda^u = \frac{10H^2 L_u^{POL}}{3 \sum_{i=1}^3 k_i^3} \frac{1}{Y_T c_T^3} \sum_{q=1}^6 \mathcal{Y}_q \left\{ [\gamma_\lambda^{(q)}]^u(\vec{k}_1, \vec{k}_2, \vec{k}_3) \mathcal{K}^{(q)}(k_1, k_2, k_3) + [\gamma_\lambda^{(q)}]^u(\vec{k}_2, \vec{k}_1, \vec{k}_3) \mathcal{K}^{(q)}(k_2, k_1, k_3) \right\} \quad (3.67)$$

where in the DS limit the dimensionful coefficients  $[\gamma_\lambda^{(q)}]^u(\vec{k}_1, \vec{k}_2, \vec{k}_3)$  can be re-expressed as

$$\begin{aligned} \gamma_\lambda^{(1)} &= \frac{c_s^{-\frac{2}{3}} c_T^{-\frac{1}{3}} K}{8k_2^3} (k_1 - k_2 - k_3)(k_1 + k_2 - k_3)(k_1 - k_2 + k_3), \\ \gamma_\lambda^{(2)} &= \gamma_\lambda^{(1)}, \quad \gamma_\lambda^{(3)} = \frac{1}{k_1^2} \gamma_\lambda^{(1)}, \quad \gamma_\lambda^{(4)} = \frac{k_3^2}{k_2^2} \gamma_\lambda^{(1)}, \quad \gamma_\lambda^{(5)} = k_3^2 \gamma_\lambda^{(1)}, \quad \gamma_\lambda^{(6)} = \frac{k_3^2}{k_1^2 k_2^2} \gamma_\lambda^{(1)}. \end{aligned} \quad (3.68)$$

Most surprisingly, the above coefficients are independent of  $\lambda$  due to no parity violation.

In the *BDS* limit we have:

$$\begin{aligned} [f_{NL;3}^{local}]_\lambda^u &= \frac{20L_u^{POL} \delta_{\lambda\lambda'}}{3 \sum_{i=1}^3 k_i^3} \frac{K^{4\nu_s+2\nu_T-9} [Cos([\nu_s - \frac{1}{2}]\frac{\pi}{2})]^{\frac{2}{3}} [Cos([\nu_T - \frac{1}{2}]\frac{\pi}{2})]^{\frac{1}{3}}}{c_s^{4\nu_s-6} c_T^{2\nu_T-3} (k_1 k_2)^{\nu_s} k_3^{\nu_T}} \\ &\quad \times \left( 2^{4\nu_s+2\nu_T-12} \left| \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right|^4 \left| \frac{\Gamma(\nu_T)}{\Gamma(\frac{3}{2})} \right|^2 \frac{(1 - \epsilon_V - s_V^T)^4 (1 - \epsilon_V - s_V^T)^2 V^{\frac{3}{2}}(\phi)}{Y_S^2 Y_T c_s^6 c_T^3 \tilde{g}_1^{\frac{3}{2}} M_{PL}^3} \right) \left( \sum_{v=1}^6 \mathcal{Y}_v(\hat{\nabla}_v)_{\lambda'} \right) \end{aligned} \quad (3.69)$$

where in the BDS limit we have:

$$\begin{aligned} (\hat{\nabla}_1)_{\lambda'} &= \left[ \frac{(k_2^{\lambda'} k_3^{\lambda''} + k_3^{\lambda'} k_2^{\lambda''})}{k_1^{\nu_T} (k_2 k_3)^{\nu_s}} + \frac{(k_1^{\lambda'} k_3^{\lambda''} + k_3^{\lambda'} k_1^{\lambda''})}{k_2^{\nu_T} (k_1 k_3)^{\nu_s}} + \frac{(k_1^{\lambda'} k_2^{\lambda''} + k_2^{\lambda'} k_1^{\lambda''})}{k_3^{\nu_T} (k_1 k_2)^{\nu_s}} \right] \tilde{O}_{\delta_{\lambda', \lambda''}}, \\ \frac{(\hat{\nabla}_2)_{\lambda'}}{c_s(\frac{3}{2} - \nu_T)} &= \left[ \frac{(k_2^{\lambda'} k_3^{\lambda''} P_{123} + k_3^{\lambda'} k_2^{\lambda''} P_{132})}{k_1^{\nu_T} (k_2 k_3)^{\nu_s}} + \frac{(k_1^{\lambda'} k_3^{\lambda''} P_{213} + k_3^{\lambda'} k_1^{\lambda''} P_{231})}{k_2^{\nu_T} (k_1 k_3)^{\nu_s}} + \frac{(k_1^{\lambda'} k_2^{\lambda''} P_{312} + k_2^{\lambda'} k_1^{\lambda''} P_{321})}{k_3^{\nu_T} (k_1 k_2)^{\nu_s}} \right] \delta_{\lambda', \lambda''}, \\ (\hat{\nabla}_3)_{\lambda'} &= c_s \left[ \frac{(k_2^{\lambda'} k_3^{\lambda''} R_{123} + k_3^{\lambda'} k_2^{\lambda''} R_{132})}{k_1^{\nu_T} (k_2 k_3)^{\nu_s}} + \frac{(k_1^{\lambda'} k_3^{\lambda''} R_{213} + k_3^{\lambda'} k_1^{\lambda''} R_{231})}{k_2^{\nu_T} (k_1 k_3)^{\nu_s}} + \frac{(k_1^{\lambda'} k_2^{\lambda''} R_{312} + k_2^{\lambda'} k_1^{\lambda''} R_{321})}{k_3^{\nu_T} (k_1 k_2)^{\nu_s}} \right] \delta_{\lambda', \lambda''}, \\ (\hat{\nabla}_4)_{\lambda'} &= \left[ k_1^2 \frac{(k_2^{\lambda'} k_3^{\lambda''} \tilde{R}_{123} + k_3^{\lambda'} k_2^{\lambda''} \tilde{R}_{132})}{k_1^{\nu_T} (k_2 k_3)^{\nu_s}} + k_2^2 \frac{(k_1^{\lambda'} k_3^{\lambda''} \tilde{R}_{213} + k_3^{\lambda'} k_1^{\lambda''} \tilde{R}_{231})}{k_2^{\nu_T} (k_1 k_3)^{\nu_s}} + k_3^2 \frac{(k_1^{\lambda'} k_2^{\lambda''} \tilde{R}_{312} + k_2^{\lambda'} k_1^{\lambda''} \tilde{R}_{321})}{k_3^{\nu_T} (k_1 k_2)^{\nu_s}} \right] \delta_{\lambda', \lambda''}, \\ (\hat{\nabla}_5)_{\lambda'} &= \left[ k_1^2 \frac{(k_2^{\lambda'} k_3^{\lambda''} + k_3^{\lambda'} k_2^{\lambda''})}{k_1^{\nu_T} (k_2 k_3)^{\nu_s}} + k_2^2 \frac{(k_1^{\lambda'} k_3^{\lambda''} + k_3^{\lambda'} k_1^{\lambda''})}{k_2^{\nu_T} (k_1 k_3)^{\nu_s}} + k_3^2 \frac{(k_1^{\lambda'} k_2^{\lambda''} + k_2^{\lambda'} k_1^{\lambda''})}{k_3^{\nu_T} (k_1 k_2)^{\nu_s}} \right] \tilde{O}_{\delta_{\lambda', \lambda''}}, \\ (\hat{\nabla}_6)_{\lambda'} &= \left[ k_1^2 \frac{(k_2^{\lambda'} k_3^{\lambda''} \tilde{L}_{123} + k_3^{\lambda'} k_2^{\lambda''} \tilde{L}_{132})}{k_1^{\nu_T} (k_2 k_3)^{\nu_s}} + k_2^2 \frac{(k_1^{\lambda'} k_3^{\lambda''} \tilde{L}_{213} + k_3^{\lambda'} k_1^{\lambda''} \tilde{L}_{231})}{k_2^{\nu_T} (k_1 k_3)^{\nu_s}} + k_3^2 \frac{(k_1^{\lambda'} k_2^{\lambda''} \tilde{L}_{312} + k_2^{\lambda'} k_1^{\lambda''} \tilde{L}_{321})}{k_3^{\nu_T} (k_1 k_2)^{\nu_s}} \right] \delta_{\lambda', \lambda''}, \end{aligned} \quad (3.70)$$

where  $k_i^\lambda = k_i$  where  $i = 1, 2, 3$ .



In the equilateral limit the expression for the non-Gaussian parameter ( $f_{NL}$ ) reduces to the following form:

$$\left[ [f_{NL;3}^{local}]_{\lambda}^{u;equil} \right]_{DS} = \frac{20H^2 L_u^{POL}}{9k^3} \frac{1}{Y_T c_T^3} \sum_{q=1}^6 \mathcal{Y}_q \left[ \gamma_{\lambda}^{(q)} \right]^u (\vec{k}, \vec{k}, \vec{k}) \mathcal{K}^{(q)}(k, k, k) \quad (3.71)$$

$$\begin{aligned} \left[ [f_{NL;3}^{local}]_{\lambda}^{u;equil} \right]_{BDS} &= \frac{20L_u^{POL} \delta_{\lambda\lambda'}}{9k^3} \frac{((2c_s + c_T)k)^{4\nu_s + 2\nu_T - 9} \left[ \text{Cos} \left( \left[ \nu_s - \frac{1}{2} \right] \frac{\pi}{2} \right) \right]^{\frac{2}{3}} \left[ \text{Cos} \left( \left[ \nu_T - \frac{1}{2} \right] \frac{\pi}{2} \right) \right]^{\frac{1}{3}}}{c_s^{4\nu_s - 6} c_T^{2\nu_T - 3} k^{2\nu_s + \nu_T}} \\ &\times \left( 2^{4\nu_s + 2\nu_T - 12} \left| \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right|^4 \left| \frac{\Gamma(\nu_T)}{\Gamma(\frac{3}{2})} \right|^2 \frac{(1 - \epsilon_V - s_V^S)^4 (1 - \epsilon_V - s_V^T)^2 V^{\frac{3}{2}}(\phi)}{Y_S^2 Y_T c_s^6 c_T^3 \tilde{g}_1^{\frac{3}{2}} M_{PL}^3} \right) \left( \sum_{v=1}^6 \mathcal{Y}_v \left( \hat{\nabla}_v \right)_{\lambda'}^{equil} \right) \end{aligned} \quad (3.72)$$

#### D. Three tensor correlation

The interactions involving three tensors are given by the following third order perturbative action:

$$\left( S^{(4)} \right)_{hhh} = \int dt d^3x a^3 \left\{ \frac{\sigma}{12} \dot{h}_{ij} \dot{h}_{jk} \dot{h}_{ki} + \frac{Y_T}{4a^2 c_T^2} \left( h_{ik} h_{jl} - \frac{1}{2} h_{ij} h_{kl} \right) h_{ij,kl} \right\} \quad (3.73)$$

Now following the prescription of *in-in formalism* in the interaction picture *three point one scalar two tensor correlation function* both for *DS* and *BDS* can be expressed in the following form:

$$\begin{aligned} \langle h_{i_1 j_1}(\vec{k}_1) h_{i_2 j_2}(\vec{k}_2) h_{i_3 j_3}(\vec{k}_3) \rangle &= -i \int_{-\infty}^0 d\eta a \langle 0 | \left[ h_{i_1 j_1}(\vec{k}_1) h_{i_2 j_2}(\vec{k}_2) h_{i_3 j_3}(\vec{k}_3), \left( [H_{int}(\eta)]_{i_1 j_1 i_2 j_2 i_3 j_3} \right)_{hhh} \right] | 0 \rangle \\ &= (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \{ B_{hhh} \}_{i_1 j_1 i_2 j_2 i_3 j_3}(\vec{k}_1, \vec{k}_2, \vec{k}_3), \end{aligned} \quad (3.74)$$

where the total Hamiltonian in the interaction picture expressed in terms of the third order Lagrangian density as  $\left( [H_{int}(\eta)]_{i_1 j_1 i_2 j_2 i_3 j_3} \right)_{hhh} = - \int d^3x [(\mathcal{L}_3)_{hhh}]_{i_1 j_1 i_2 j_2 i_3 j_3}$ . In this context the bispectrum for three tensor correlation can be expressed as:

$$\{ B_{hhh} \}_{i_1 j_1 i_2 j_2 i_3 j_3}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{(2\pi)^4 \mathcal{P}_u^2}{\prod_{i=1}^3 k_i^3} \mathcal{A}_{i_1 j_1 i_2 j_2 i_3 j_3}^{hhh} = \frac{6}{5} [f_{NL;4}^{local}] P_u^2, \quad (3.75)$$

where the symbol ; 4 represents three tensor correlation. In this context the non-Gaussian parameter for *DS* and *BDS* limit can be expressed as:

$$\begin{aligned} \left[ [f_{NL;4}^{local}]_{i_1 j_1 i_2 j_2 i_3 j_3} \right]_{DS} &= \frac{10\mathcal{W}_u^{POL}}{3 \sum_{i=1}^3 k_i^3} \left[ \mathcal{A}_{i_1 j_1 i_2 j_2 i_3 j_3}^{hhh}(\vec{k}_1, \vec{k}_2, \vec{k}_3) + \mathcal{A}_{i_1 j_1 i_2 j_2 i_3 j_3}^{hhh}(\vec{k}_1, \vec{k}_3, \vec{k}_2) + \mathcal{A}_{i_1 j_1 i_2 j_2 i_3 j_3}^{hhh}(\vec{k}_3, \vec{k}_2, \vec{k}_1) \right. \\ &\quad \left. + \mathcal{A}_{i_1 j_1 i_2 j_2 i_3 j_3}^{hhh}(\vec{k}_3, \vec{k}_1, \vec{k}_2) + \mathcal{A}_{i_1 j_1 i_2 j_2 i_3 j_3}^{hhh}(\vec{k}_2, \vec{k}_1, \vec{k}_3) + \mathcal{A}_{i_1 j_1 i_2 j_2 i_3 j_3}^{hhh}(\vec{k}_2, \vec{k}_3, \vec{k}_1) \right] \end{aligned} \quad (3.76)$$

where in *DS* limit we define

$$\begin{aligned} \mathcal{A}_{i_1 j_1 i_2 j_2 i_3 j_3}^{hhh}(\vec{k}_1, \vec{k}_2, \vec{k}_3) &:= \frac{H\sigma}{4Y_T} \frac{k_1^2 k_2^2 k_3^2}{K^3} \sum_{l,m,n} \mathcal{N}_{i_1 j_1, lm}(\vec{k}_1) \mathcal{N}_{i_2 j_2, mn}(\vec{k}_2) \mathcal{N}_{i_3 j_3, nl}(\vec{k}_3) \\ &\quad - \frac{K}{16} \left[ 1 - \frac{1}{K^3} \sum_{i \neq j} k_i^2 k_j^2 - 4 \frac{k_1 k_2 k_3}{K^3} \right] \left\{ \mathcal{N}_{i_1 j_1, ik}(\vec{k}_1) \mathcal{N}_{i_2 j_2, jl}(\vec{k}_2) \right. \\ &\quad \left. \times \left[ k_{3k} k_{3l} \mathcal{N}_{i_3 j_3, ij}(\vec{k}_3) - \frac{1}{2} k_{3i} k_{3k} \mathcal{N}_{i_3 j_3, jl}(\vec{k}_3) \right] \right\}. \end{aligned} \quad (3.77)$$

In the *BDS* limit

$$\begin{aligned} \left[ [f_{NL;4}^{local}]_{i_1 j_1 i_2 j_2 i_3 j_3} \right]_{BDS} &= \frac{10\mathcal{W}_u^{POL}}{3 \sum_{i=1}^3 k_i^3} \frac{K^{9-6\nu_T} \text{Cos} \left( \left[ \nu_T - \frac{1}{2} \right] \frac{\pi}{2} \right)}{(k_1 k_2 k_3)^{2\nu_T}} \left( 2^{3(\nu_s + \nu_T) - 11} \left| \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right|^3 \left| \frac{\Gamma(\nu_T)}{\Gamma(\frac{3}{2})} \right|^3 \right. \\ &\quad \left. \frac{(1 - \epsilon_V - s_V^S)^3 (1 - \epsilon_V - s_V^T)^2 V^{\frac{3}{2}}(\phi)}{Y_S^{\frac{3}{2}} Y_T^{\frac{3}{2}} c_s^{\frac{9}{2}} c_T^{\frac{9}{2}} \tilde{g}_1^{\frac{3}{2}} M_{PL}^3} \right) \left( \sum_{p=1}^3 \Delta_{i_1 j_1 i_2 j_2 i_3 j_3}^{(p)} \right) \end{aligned} \quad (3.78)$$

where  $K = k_1 + k_2 + k_3$  and the polarization index  $u = 1(E - mode), 2(E \otimes B - mode), 3(B - mode)$ . In BDS limit we have:

$$\begin{aligned}\Delta_{i_1 j_1 i_2 j_2 i_3 j_3}^{(1)} &= \frac{\sigma}{12} \mathcal{N}_{i_1 j_1; i j} \mathcal{N}_{i_2 j_2; j k} \mathcal{N}_{i_3 j_3; k i} c_s^3 \left( \frac{3}{2} - \nu_T \right)^3 [M_{123} + M_{132} + M_{213} + M_{231} + M_{312} + M_{321}], \\ \Delta_{i_1 j_1 i_2 j_2 i_3 j_3}^{(2)} &= \frac{Y_T}{2c_T^2} \mathcal{N}_{i_1 j_1; i k} \mathcal{N}_{i_2 j_2; j l} \mathcal{N}_{i_3 j_3; i j} [k_{3k} k_{3l} + k_{2k} k_{2l} + k_{1k} k_{1l}] \mathcal{Q}, \\ \Delta_{i_1 j_1 i_2 j_2 i_3 j_3}^{(3)} &= -\frac{Y_T}{2c_T^2} \mathcal{N}_{i_1 j_1; i_3 j_3} \mathcal{N}_{i_2 j_2; k l} [k_{3k} k_{3l} + k_{2k} k_{2l} + k_{1k} k_{1l}] \mathcal{Q}\end{aligned}\quad (3.79)$$

with

$$\mathcal{Q} = \sum_{p=1}^4 O_p \frac{\Gamma(9+p-6\nu_T)}{K^{9+p-6\nu_T}}, \quad (3.80)$$

$$\begin{aligned}M_{abc} &= \sum_{p=1}^7 b_p^{abc} \frac{\Gamma(6+p-6\nu_T)}{K^{6+p-6\nu_T}}, \\ b_1^{abc} &= \left( \frac{3}{2} - \nu_T \right)^3, b_2^{abc} = b_1^{abc} K, \\ b_3^{abc} &= \left( \frac{3}{2} - \nu_T \right)^2 [k_a(k_b + k_c) + k_b^2 + k_c^2 + k_b k_c + k_a^2], \\ b_4^{abc} &= \left[ k_a^2(k_b + k_c) \left( \frac{3}{2} - \nu_T \right) + \{k_a(k_b^2 + k_c^2 + k_b k_c) + k_b k_c(k_b + k_c)\} \left( \frac{3}{2} - \nu_T \right)^2 \right], \\ b_5^{abc} &= \left[ \left( \frac{3}{2} - \nu_T \right) \{k_a^2(k_b^2 + k_c^2 + k_b k_c) + k_b^2 k_c^2\} + \left( \frac{3}{2} - \nu_T \right)^2 k_a k_b k_c(k_b + k_c) \right], \\ b_6^{abc} &= \left( \frac{3}{2} - \nu_T \right) k_a k_b k_c [k_b k_c + k_a(k_b + k_c)], b_7^{abc} = -k_a^2 k_b^2 k_c^2.\end{aligned}$$

For both  $DS$  and  $BDS$  limit the overall normalization factor for three types of polarization can be expressed as:

$$\mathcal{W}_u^{POL} = \begin{cases} 4 & :u=1(E-mode) \\ 64 & :u=2(E \otimes B mode) \\ 1024 & :u=3(B-mode). \end{cases} \quad (3.81)$$

After performing basis transformation the relevant three point correlation function for three tensor interaction can be expressed in terms of bispectrum as:

$$\langle \bigoplus^{\lambda_1}(\vec{k}_1) \bigoplus^{\lambda_2}(\vec{k}_2) \bigoplus^{\lambda_3}(\vec{k}_3) \rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_{\lambda_1, \lambda_2, \lambda_3}^{hhh}. \quad (3.82)$$

where

$$B_{\lambda_1, \lambda_2, \lambda_3}^{hhh} = \frac{(2\pi)^4 \mathcal{P}_u^2}{\prod_{i=1}^3 k_i^3} \mathcal{A}_{(\zeta \zeta h)}^{\lambda_1, \lambda_2, \lambda_3} = \frac{6}{5} [f_{NL;4}^{local}]_{\lambda_1, \lambda_2, \lambda_3}^u P_u^2, \quad (3.83)$$

where the non-linear parameter is given by:

$$\begin{aligned}\left[ [f_{NL;4}^{local}]_{\lambda_1, \lambda_2, \lambda_3}^u \right]_{DS} &= \frac{10 \mathcal{W}_u^{POL}}{3 \sum_{i=1}^3 k_i^3} F(\lambda_1 k_1, \lambda_2 k_2, \lambda_3 k_3) \left[ \frac{H\sigma}{4Y_T} \frac{\prod_{i=1}^3 k_i^2}{K^3} \right. \\ &\quad \left. + \frac{K}{32} \left[ 1 - \frac{1}{K^3} \sum_{i \neq j} k_i^2 k_j - 4 \frac{k_1 k_2 k_3}{K^3} \right] (\lambda_1 k_1 + \lambda_2 k_2 + \lambda_3 k_3)^2 \right]\end{aligned}\quad (3.84)$$

where in  $DS$  limit we define a new function as:

$$F(\alpha, \beta, \rho) := \frac{1}{64} \frac{1}{\alpha^2 \beta^2 \rho^2} (\alpha + \beta + \rho)^3 (\alpha - \beta + \rho) (\alpha + \beta - \rho) (\alpha - \beta - \rho). \quad (3.85)$$

Similarly in the *BDS* limit

$$\left[ [f_{NL;4}^{local}]_{\lambda_1, \lambda_2, \lambda_3}^u \right]_{BDS} = \frac{10\mathcal{W}_u^{POL}}{3 \sum_{i=1}^3 k_i^3} \frac{K^{9-6\nu_T} \text{Cos} \left( \left[ \nu_T - \frac{1}{2} \right] \frac{\pi}{2} \right)}{(k_1 k_2 k_3)^{2\nu_T}} \left( 2^{3(\nu_s + \nu_T) - 11} \left| \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right|^3 \left| \frac{\Gamma(\nu_T)}{\Gamma(\frac{3}{2})} \right|^3 \right. \\ \left. \frac{(1 - \epsilon_V - s_V^S)^3 (1 - \epsilon_V - s_V^T)^3 V^{\frac{3}{2}}(\phi)}{Y_S^{\frac{3}{2}} Y_T^{\frac{3}{2}} c_s^{\frac{9}{2}} c_T^{\frac{9}{2}} \tilde{g}_1^{\frac{3}{2}} M_{PL}^3} \right) \left( \sum_{p=1}^3 \Delta_{\lambda_1 \lambda_2 \lambda_3}^{(p)} \right) \quad (3.86)$$

where in *BDS* limit the helicity dependent functions are given by:

$$\begin{aligned} \Delta_{\lambda_1 \lambda_2 \lambda_3}^{(1)} &= \frac{\sigma}{12} \delta_{\lambda_1 \lambda'} \delta_{\lambda_2 \lambda''} \delta_{\lambda_3 \lambda'''} c_s^3 \left( \frac{3}{2} - \nu_T \right)^3 \left[ M_{123}^{\lambda' \lambda'' \lambda'''} + M_{132}^{\lambda' \lambda'' \lambda'''} + M_{213}^{\lambda' \lambda'' \lambda'''} + M_{231}^{\lambda' \lambda'' \lambda'''} + M_{312}^{\lambda' \lambda'' \lambda'''} + M_{321}^{\lambda' \lambda'' \lambda'''} \right], \\ \Delta_{\lambda_1 \lambda_2 \lambda_3}^{(2)} &= \frac{Y_T}{2c_T^2} \delta_{\lambda_1 \lambda'} \delta_{\lambda_2 \lambda''} \delta_{\lambda_3 \lambda'''} \left[ k_3^{\lambda'} k_3^{\lambda''} + k_2^{\lambda'} k_2^{\lambda''} + k_1^{\lambda'} k_1^{\lambda''} \right] \mathcal{Q}^{\lambda'''}, \\ \Delta_{\lambda_1 \lambda_2 \lambda_3}^{(3)} &= -\frac{Y_T}{2c_T^2} \delta_{\lambda'' \lambda'} \delta_{\lambda_2 \lambda_1} \delta_{\lambda_3 \lambda'''} \left[ k_3^{\lambda'''} k_3^{\lambda''} + k_2^{\lambda'''} k_2^{\lambda''} + k_1^{\lambda'''} k_1^{\lambda''} \right] \mathcal{Q}, \end{aligned} \quad (3.87)$$

with

$$\begin{aligned} M_{abc}^{\lambda' \lambda'' \lambda'''} &= \sum_{p=1}^7 (b_p^{abc})^{\lambda' \lambda'' \lambda'''} \frac{\Gamma(6+p-6\nu_T)}{K^{6+p-6\nu_T}}, \quad \mathcal{Q}^{\lambda'''} = \sum_{p=1}^4 O_p^{\lambda'''} \frac{\Gamma(9+p-6\nu_T)}{K^{9+p-6\nu_T}}, \\ (b_1^{abc})^{\lambda' \lambda'' \lambda'''} &= \left( \frac{3}{2} - \nu_T \right)^3, \quad (b_2^{abc})^{\lambda' \lambda'' \lambda'''} = (b_1^{abc})^{\lambda' \lambda'' \lambda'''} K, \\ (b_3^{abc})^{\lambda' \lambda'' \lambda'''} &= \left( \frac{3}{2} - \nu_T \right)^2 \left[ k_a^{\lambda'} (k_b^{\lambda''} + k_c^{\lambda'''}) + (k_b^{\lambda''})^2 + (k_c^{\lambda'''})^2 + k_b^{\lambda''} k_c^{\lambda'''} + (k_a^{\lambda'})^2 \right], \\ (b_4^{abc})^{\lambda' \lambda'' \lambda'''} &= \left[ (k_a^{\lambda'})^2 (k_b^{\lambda''} + k_c^{\lambda'''}) \left( \frac{3}{2} - \nu_T \right) + \left\{ k_a^{\lambda'} ((k_b^{\lambda''})^2 + (k_c^{\lambda'''})^2 + k_b^{\lambda''} k_c^{\lambda'''}) + k_b^{\lambda''} k_c^{\lambda'''} (k_b^{\lambda''} + k_c^{\lambda'''}) \right\} \left( \frac{3}{2} - \nu_T \right)^2 \right], \\ (b_5^{abc})^{\lambda' \lambda'' \lambda'''} &= \left[ \left( \frac{3}{2} - \nu_T \right) \left\{ (k_a^{\lambda'})^2 ((k_b^{\lambda''})^2 + (k_c^{\lambda'''})^2 + k_b^{\lambda''} k_c^{\lambda'''}) + (k_b^{\lambda''} k_c^{\lambda'''})^2 \right\} + \left( \frac{3}{2} - \nu_T \right)^2 k_a^{\lambda'} k_b^{\lambda''} k_c^{\lambda'''} (k_b^{\lambda''} + k_c^{\lambda'''}) \right], \\ (b_6^{abc})^{\lambda' \lambda'' \lambda'''} &= \left( \frac{3}{2} - \nu_T \right) k_a^{\lambda'} k_b^{\lambda''} k_c^{\lambda'''} \left[ k_b^{\lambda''} k_c^{\lambda'''} + k_a^{\lambda'} (k_b^{\lambda''} + k_c^{\lambda'''}) \right], \quad (b_7^{abc})^{\lambda' \lambda'' \lambda'''} = -(k_a^{\lambda'} k_b^{\lambda''} k_c^{\lambda'''})^2, \\ O_1^{\lambda'''} &= 1, O_2^{\lambda'''} = iK^{\lambda'''}, O_3^{\lambda'''} = -(k_a^{\lambda'''} k_b + k_b^{\lambda'''} k_c + k_c^{\lambda'''} k_a), O_4^{\lambda'''} = -ik_a^{\lambda'''} k_b k_c, \end{aligned} \quad (3.88)$$

where  $k_1^{\lambda'} = \lambda' k_1$ ,  $k_2^{\lambda''} = \lambda'' k_2$  and  $k_3^{\lambda'''} = \lambda''' k_3$ . In the equilateral limit we have:

$$\left[ [f_{NL;4}^{local}]_{\lambda_1, \lambda_2, \lambda_3}^u \right]_{DS} = \frac{5\mathcal{W}_u^{POL}}{31104(\lambda_1 \lambda_2 \lambda_3)^2} \left[ \frac{H\sigma}{Y_T} + \frac{51}{8}(\lambda_1 + \lambda_2 + \lambda_3)^2 \right] \quad (3.89)$$

$$\left[ [f_{NL;4}^{local}]_{\lambda_1, \lambda_2, \lambda_3}^u \right]_{BDS} = \frac{10\mathcal{W}_u^{POL}}{9k^3} \frac{(3k)^{9-6\nu_T} \text{Cos} \left( \left[ \nu_T - \frac{1}{2} \right] \frac{\pi}{2} \right)}{k^{6\nu_T}} \left( 2^{3(\nu_s + \nu_T) - 11} \left| \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right|^3 \left| \frac{\Gamma(\nu_T)}{\Gamma(\frac{3}{2})} \right|^3 \right. \\ \left. \frac{(1 - \epsilon_V - s_V^S)^3 (1 - \epsilon_V - s_V^T)^3 V^{\frac{3}{2}}(\phi)}{Y_S^{\frac{3}{2}} Y_T^{\frac{3}{2}} c_s^{\frac{9}{2}} c_T^{\frac{9}{2}} \tilde{g}_1^{\frac{3}{2}} M_{PL}^3} \right) \left( \sum_{p=1}^3 \Delta_{\lambda_1 \lambda_2 \lambda_3}^{(p);equil} \right) \quad (3.90)$$

Numerical values of all such non-Gaussian parameters from three point correlation for different polarizing modes are mentioned in the table(I). In this context **PC** and **PV** stands for the parity conserving and violating contribution for graviton degrees of freedom.

#### IV. TREE LEVEL TRISPECTRUM ANALYSIS FROM FOUR SCALAR CORRELATION

To derive the expression for scalar trispectrum for D3 DBI Galileon let us start from fourth order perturbative action up to total derivatives. Consequently the fourth order perturbative action in the uniform gauge can be expressed as:

$$S^{(4)} = \int dt d^3x \frac{a^3}{4} \left\{ \bar{U}_1 \dot{\zeta}^4 - \frac{(\partial \zeta)^2}{a^2} \dot{\zeta}^2 \bar{U}_2 + \bar{U}_3 \frac{(\partial \zeta)^4}{a^4} \right\}, \quad (4.1)$$

where the co-efficients  $\bar{\mathcal{U}}_i (i = 1, 2, 3)$  defined as:

$$\begin{aligned}\bar{\mathcal{U}}_1 &= \left( \dot{\phi}^2 \hat{K}_{XXX} + \frac{\hat{K}_{XX}}{2} \right), \\ \bar{\mathcal{U}}_2 &= \left( \dot{\phi}^2 \hat{K}_{XXX} + \hat{K}_{XX} \right), \\ \bar{\mathcal{U}}_3 &= \frac{\hat{K}_{XX}}{2}.\end{aligned}\tag{4.2}$$

Using in-in procedure the *four point correlation function* both for *DS* and *BDS* can be expressed in the following form:

$$\begin{aligned}\langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3) \zeta(\vec{k}_4) \rangle &= -i \sum_{j=1}^3 \int_{-\infty}^0 d\eta \ a \ \langle 0 | \left[ \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3) \zeta(\vec{k}_4), \left( H_{int}^{(j)}(\eta) \right)_{\zeta\zeta\zeta\zeta} \right] | 0 \rangle \\ &= (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) \mathcal{T}_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4),\end{aligned}\tag{4.3}$$

where in the interaction picture the total Hamiltonian can be written in terms of the fourth order Lagrangian density as:  $(H_{int}(\eta))_{\zeta\zeta\zeta\zeta} = \sum_{j=1}^3 (H_{int}^{(j)}(\eta))_{\zeta\zeta\zeta\zeta} = - \int d^3x \mathcal{L}_4$ . Here the *trispectrum*  $\mathcal{T}_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)$  is defined as:

$$\begin{aligned}\mathcal{T}_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) &= \frac{(2\pi)^6 P_\zeta^3}{\prod_{i=1}^4 k_i^3} \mathcal{M}_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) \\ &= \frac{1}{\prod_{i=1}^4 k_i^3} [(k_1^3 k_2^3 + k_3^3 k_4^3) (k_{13}^{-3} + k_{14}^{-3}) + (k_1^3 k_4^3 + k_2^3 k_3^3) (k_{12}^{-3} + k_{13}^{-3}) \\ &\quad + (k_1^3 k_3^3 + k_2^3 k_4^3) (k_{12}^{-3} + k_{14}^{-3})] \left\{ \tau_{NL}^{local} P_{\zeta(1)}^3 + \frac{54}{25} g_{NL}^{local} P_{\zeta(2)}^3 \right\},\end{aligned}\tag{4.4}$$

where  $\mathcal{M}_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)$  is the *shape function* for trispectrum and

$$\begin{aligned}P_{\zeta(1)}^3 &= P_\zeta(k_{13}) P_\zeta(k_3) P_\zeta(k_4) + P_\zeta(k_{13}) P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_{24}) P_\zeta(k_2) P_\zeta(k_1) + P_\zeta(k_{24}) P_\zeta(k_3) P_\zeta(k_4) \\ &\quad + P_\zeta(k_{23}) P_\zeta(k_2) P_\zeta(k_1) + P_\zeta(k_{23}) P_\zeta(k_4) P_\zeta(k_3) + P_\zeta(k_{14}) P_\zeta(k_4) P_\zeta(k_3) + P_\zeta(k_{14}) P_\zeta(k_1) P_\zeta(k_2) \\ &\quad + P_\zeta(k_{12}) P_\zeta(k_1) P_\zeta(k_4) + P_\zeta(k_{12}) P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_{34}) P_\zeta(k_3) P_\zeta(k_2) + P_\zeta(k_{34}) P_\zeta(k_4) P_\zeta(k_1), \\ P_{\zeta(2)}^3 &= P_\zeta(k_2) P_\zeta(k_3) P_\zeta(k_4) + P_\zeta(k_1) P_\zeta(k_3) P_\zeta(k_4) + P_\zeta(k_1) P_\zeta(k_2) P_\zeta(k_4) + P_\zeta(k_1) P_\zeta(k_2) P_\zeta(k_3)\end{aligned}\tag{4.5}$$

such that

$$\begin{aligned}P_\zeta^3 &= P_{\zeta(1)}^3 + \frac{3456}{25} \frac{1}{\bar{K}^3} [(k_1^3 k_2^3 + k_3^3 k_4^3) (k_{13}^{-3} + k_{14}^{-3}) \\ &\quad + (k_1^3 k_4^3 + k_2^3 k_3^3) (k_{12}^{-3} + k_{13}^{-3}) + (k_1^3 k_3^3 + k_2^3 k_4^3) (k_{12}^{-3} + k_{14}^{-3})] P_{\zeta(2)}^3\end{aligned}\tag{4.6}$$

and  $\tau_{NL}^{local}$  and  $g_{NL}^{local}$  are the two non linear parameters which carry the signatures of primordial non-Gaussianities of the curvature perturbation in trispectrum analysis. By knowing  $\tau_{NL}^{local}$  the other parameter  $g_{NL}^{local}$  can be calculated by making use of the following relation:

$$\begin{aligned}g_{NL}^{local} &= \frac{64}{\bar{K}^3} [(k_1^3 k_2^3 + k_3^3 k_4^3) (k_{13}^{-3} + k_{14}^{-3}) + (k_1^3 k_4^3 + k_2^3 k_3^3) (k_{12}^{-3} + k_{13}^{-3}) + (k_1^3 k_3^3 \\ &\quad + k_2^3 k_4^3) (k_{12}^{-3} + k_{14}^{-3})] \tau_{NL}^{local},\end{aligned}\tag{4.7}$$

where  $\bar{K} = k_1 + k_2 + k_3 + k_4$ . So, there is only one independent piece of information, namely  $\tau_{NL}^{local}$ , that carries information about trispectrum.

To proceed further we denote the angle between  $\vec{k}_i$  and  $\vec{k}_j$  (with  $i \neq j$ ) by  $\Theta_{ij}$  then

$$\begin{aligned}\cos(\Theta_{12}) &= \cos(\Theta_{34}) := \cos(\Theta_3), \\ \cos(\Theta_{23}) &= \cos(\Theta_{14}) := \cos(\Theta_1), \\ \cos(\Theta_{13}) &= \cos(\Theta_{24}) := \cos(\Theta_2)\end{aligned}\tag{4.8}$$

subject to the constraint

$$\cos(\Theta_1) + \cos(\Theta_2) + \cos(\Theta_3) = -1\tag{4.9}$$

comes from the conservation of momentum.

The explicit form of  $\tau_{NL}^{local}$  characterizing the trispectrum can be expressed for our model as:

$$\begin{aligned}
[\tau_{NL}^{local}]_{DS} = & \frac{64\pi^6}{\left[ (k_1^3 k_2^3 + k_3^3 k_4^3) (k_{13}^{-3} + k_{14}^{-3}) + (k_1^3 k_4^3 + k_2^3 k_3^3) (k_{12}^{-3} + k_{13}^{-3}) + (k_1^3 k_3^3 + k_2^3 k_4^3) (k_{12}^{-3} + k_{14}^{-3}) \right]} \\
& \times \left\{ \frac{9\bar{U}_1 \epsilon_V^3 H^2}{2Y_S^4 c_s^6} \frac{\prod_{i=1}^4 k_i^2}{\bar{K}^5} + \frac{\bar{U}_2 H^2}{16Y_S^4 c_s^8} \left[ \frac{k_1^2 k_2^2 (\vec{k}_3 \cdot \vec{k}_4)}{\bar{K}^3} \left( 1 + \frac{3(k_3 + k_4)}{\bar{K}} + \frac{12k_3 k_4}{\bar{K}^2} \right) \right. \right. \\
& + \frac{k_3^2 k_1^2 (\vec{k}_2 \cdot \vec{k}_4)}{\bar{K}^3} \left( 1 + \frac{3(k_2 + k_4)}{\bar{K}} + \frac{12k_2 k_4}{\bar{K}^2} \right) + \frac{k_1^2 k_4^2 (\vec{k}_2 \cdot \vec{k}_3)}{\bar{K}^3} \left( 1 + \frac{3(k_2 + k_3)}{\bar{K}} + \frac{12k_2 k_3}{\bar{K}^2} \right) \\
& + \frac{k_2^2 k_3^2 (\vec{k}_1 \cdot \vec{k}_4)}{\bar{K}^3} \left( 1 + \frac{3(k_1 + k_4)}{\bar{K}} + \frac{12k_1 k_4}{\bar{K}^2} \right) + \frac{k_2^2 k_4^2 (\vec{k}_3 \cdot \vec{k}_1)}{\bar{K}^3} \left( 1 + \frac{3(k_3 + k_1)}{\bar{K}} + \frac{12k_3 k_1}{\bar{K}^2} \right) \\
& + \frac{k_3^2 k_4^2 (\vec{k}_1 \cdot \vec{k}_2)}{\bar{K}^3} \left( 1 + \frac{3(k_1 + k_2)}{\bar{K}} + \frac{12k_1 k_2}{\bar{K}^2} \right) \left. \right] + \frac{\bar{U}_3 H^2}{8Y_S^4 c_s^{10} \bar{K}} \left[ (\vec{k}_1 \cdot \vec{k}_2)(\vec{k}_3 \cdot \vec{k}_4) + (\vec{k}_1 \cdot \vec{k}_3)(\vec{k}_2 \cdot \vec{k}_4) \right. \\
& \left. \left. + (\vec{k}_1 \cdot \vec{k}_4)(\vec{k}_2 \cdot \vec{k}_3) \right] \left( 1 + \frac{\sum_{i,j(i<j)=1}^4 k_i k_j}{\bar{K}} + \frac{3k_1 k_2 k_3 k_4}{\bar{K}^3} \left( \sum_{i=1}^4 \frac{1}{k_i} \right) + \frac{12k_1 k_2 k_3 k_4}{\bar{K}^4} \right) \right\}
\end{aligned} \tag{4.10}$$

$$\begin{aligned}
[\tau_{NL}^{local}]_{BDS} = & \frac{2^{8\nu_s-6} \pi^6 \text{Cos} \left( \left[ \nu_s - \frac{1}{2} \right] \frac{\pi}{2} \right)}{\left[ (k_1^3 k_2^3 + k_3^3 k_4^3) (k_{13}^{-3} + k_{14}^{-3}) + (k_1^3 k_4^3 + k_2^3 k_3^3) (k_{12}^{-3} + k_{13}^{-3}) + (k_1^3 k_3^3 + k_2^3 k_4^3) (k_{12}^{-3} + k_{14}^{-3}) \right]} \\
& \times \left\{ \frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})} \right\}^8 \frac{(1 - \epsilon_V - s_V^S)^8 H^8}{Y_S^4 c_s^{12} (k_1 k_2 k_3 k_4)^{2\nu_s}} \left\{ \frac{8\bar{U}_1 \bar{K}^{8\nu_s-5}}{13} \left[ \Gamma(17 - 8\nu_s) \bar{K}^8 G_1 - i\Gamma(16 - 8\nu_s) \bar{K}^7 G_2 \right. \right. \\
& + \Gamma(15 - 8\nu_s) \bar{K}^6 G_3 - i\Gamma(14 - 8\nu_s) \bar{K}^5 G_4 + \Gamma(13 - 8\nu_s) \bar{K}^4 G_5 - i\Gamma(12 - 8\nu_s) \bar{K}^3 G_6 \\
& + \Gamma(11 - 8\nu_s) \bar{K}^2 G_7 - i\Gamma(10 - 8\nu_s) \bar{K} G_8 + G_9 \left. \right] + \frac{\bar{U}_2 \bar{K}^{8\nu_s-3}}{32} \left[ (\vec{k}_3 \cdot \vec{k}_4) \bar{\mathcal{I}}(3, 4; 1, 2) + (\vec{k}_2 \cdot \vec{k}_4) \bar{\mathcal{I}}(2, 4; 1, 3) \right. \\
& + (\vec{k}_2 \cdot \vec{k}_3) \bar{\mathcal{I}}(2, 3; 1, 4) + (\vec{k}_1 \cdot \vec{k}_4) \bar{\mathcal{I}}(1, 4; 2, 3) + (\vec{k}_1 \cdot \vec{k}_2) \bar{\mathcal{I}}(1, 2; 3, 4) + (\vec{k}_1 \cdot \vec{k}_3) \bar{\mathcal{I}}(1, 3; 2, 4) \left. \right] \\
& + \frac{\bar{U}_3 \bar{K}^{8\nu_s+12}}{8} \left[ (\vec{k}_1 \cdot \vec{k}_2)(\vec{k}_3 \cdot \vec{k}_4) + (\vec{k}_1 \cdot \vec{k}_3)(\vec{k}_2 \cdot \vec{k}_4) + (\vec{k}_1 \cdot \vec{k}_4)(\vec{k}_2 \cdot \vec{k}_3) \right] \left( \frac{\bar{\mathcal{Z}}_1 \Gamma(13 - 8\nu_s)}{(\bar{K})^{13}} \right. \\
& \left. \left. + \frac{\bar{\mathcal{Z}}_2 \Gamma(14 - 8\nu_s)}{(\bar{K})^{14}} - \frac{\bar{\mathcal{Z}}_3 \Gamma(15 - 8\nu_s)}{(\bar{K})^{15}} - \frac{\bar{\mathcal{Z}}_4 \Gamma(16 - 8\nu_s)}{(\bar{K})^{16}} + \frac{\bar{\mathcal{Z}}_5 \Gamma(17 - 8\nu_s)}{(\bar{K})^{17}} \right) \right\}
\end{aligned} \tag{4.11}$$

where in  $BDS$  limit

$$\begin{aligned}
G_1 &= \left( \frac{3}{2} - \nu_s \right)^4, \quad G_2 = i\bar{K} G_1, \quad G_3 = G_1^{\frac{3}{4}} \left( \sum_{i=1}^4 k_i^2 \right) + G_1 \left( \sum_{i,j(i>j)=1}^4 k_i k_j \right), \\
G_4 &= iG_1 \left( \sum_{i,j,m(i>j>m)=1}^4 k_i k_j k_m \right) + iG_i^{\frac{3}{4}} \left( \sum_{i,j(i \neq j)=1}^4 k_i^2 k_j \right), \\
G_5 &= \sqrt{G_1} \left( \sum_{i,j(i>j)=1}^4 k_i^2 k_j^2 \right) + G_1^{\frac{3}{4}} \left( \sum_{i,j,m(i>j>m)=1}^4 k_i^2 k_j k_m \right) + G_1 \left( \prod_{i=1}^4 k_i \right), \\
G_6 &= i\sqrt{G_1} \left( \sum_{i,j,m=1}^4 k_i^2 k_j^2 k_m \right) + i \left( \prod_{i,j,m,n(i \neq j > m > n)=1}^4 k_i^2 k_j k_m k_n \right), \\
G_7 &= G_1^{\frac{1}{4}} \left( \prod_{i,j,m(i>j>m)=1}^4 k_i^2 k_j^2 k_m^2 \right) + \sqrt{G_1} \left( \prod_{i,j,m,n(i<j,m<n,i \neq m,j \neq n)=1}^4 k_i^2 k_j^2 k_m k_n \right), \\
G_8 &= iG_1^{\frac{3}{4}} \left( \prod_{i,j,m,n(i>j>m \neq n)=1}^4 k_i^2 k_j^2 k_m^2 k_n \right), \quad G_9 = \left( \prod_{i=1}^4 k_i^2 \right), \quad \bar{\mathcal{Z}}_1 = 1, \\
\bar{\mathcal{Z}}_2 &= i\bar{K}, \quad \bar{\mathcal{Z}}_3 = \left( \prod_{i,j(i>j)=1}^4 k_i k_j \right), \quad \bar{\mathcal{Z}}_4 = \left( \prod_{i,j,m(i>j>m)=1}^4 k_i k_j k_m \right), \quad \bar{\mathcal{Z}}_5 = \left( \prod_{i=1}^4 k_i \right)
\end{aligned} \tag{4.12}$$

and

$$\begin{aligned}
\bar{\mathcal{I}}(i, j; m, n) &= \left[ \bar{K}^4 \sqrt{G_1} \Gamma(10 - 6\nu_s) + \bar{K}^3 \sqrt{G_1} (k_i + k_j + k_m + k_n) \Gamma(11 - 6\nu_s) \right. \\
&- \bar{K}^2 \left\{ k_m^2 k_n^2 - iG_1^{\frac{1}{4}} k_m k_n (k_m + k_n) - k_m k_n \sqrt{G_1} - k_i k_j \sqrt{G_1} - (k_i + k_j)(k_m + k_n) \sqrt{G_1} \right\} \Gamma(12 - 6\nu_s) \\
&\left. + \bar{K} \left\{ k_i k_j (k_m + k_n) \sqrt{G_1} - (k_i + k_j) k_m k_n \sqrt{G_1} \right\} \Gamma(13 - 6\nu_s) + k_m k_n \sqrt{G_1} \Gamma(14 - 6\nu_s) \right].
\end{aligned} \tag{4.13}$$

and in both the limit we have used

$$\begin{aligned}
k_{14} &= k_{23} = \sqrt{k_1^2 + k_4^2 + 2k_1 k_4 \text{Cos}(\Theta_1)}, \\
k_{24} &= k_{13} = \sqrt{k_2^2 + k_4^2 + 2k_2 k_4 \text{Cos}(\Theta_2)}, \\
k_{34} &= k_{12} = \sqrt{k_3^2 + k_4^2 + 2k_3 k_4 \text{Cos}(\Theta_3)}.
\end{aligned} \tag{4.14}$$

Further, using the *equilateral configuration* ( $k_1 = k_2 = k_3 = k_4 = k$  and  $\bar{K} = 4k$ ) and incorporating the contribution from the maximum shape of the trispectrum ( $\text{Cos}(\Theta_1) = \text{Cos}(\Theta_2) = \text{Cos}(\Theta_3) = -\frac{1}{3}$  and  $k_{ij}(\text{for } i < j) = \frac{2k}{\sqrt{3}}$ ) the non linear parameter  $\tau_{NL}$  can be recast as:

$$\left[\tau_{NL}^{equil}\right]_{DS} = \frac{(1 + \delta_{GX} + \mathcal{O}(\epsilon_s^2))^2 \pi^6}{48\sqrt{3}\epsilon_s^4 M_{PL}^8 L_1^2} \left\{ 3(\epsilon_s - \delta_{GX} - \mathcal{O}(\epsilon_s^2))^3 \left[ 4\left(1 - \frac{3}{4}c_s^2\right) \left\{ \frac{4\bar{\mathcal{U}}_3}{c_s^2} \left(1 - \frac{3}{2}c_s^2\right) - \bar{\mathcal{U}}_2 \right\} \right. \right. \right. \\ \left. \left. - 8\bar{\mathcal{U}}_3 \left(1 - \frac{9}{8}c_s^2\right) + \frac{\phi^2}{\bar{K}c_s^2} \left\{ \frac{4\bar{\mathcal{U}}_3}{c_s^2} \left(1 - \frac{3}{2}c_s^2\right) - \bar{\mathcal{U}}_2 \right\}^2 \right] - \frac{13}{3} + \frac{176}{9c_s^2} \right\} \quad (4.15)$$

$$\left[\tau_{NL}^{equil}\right]_{BDS} = \frac{2^{24\nu_s-5}\pi^6 \text{Cos}\left(\left[\nu_s - \frac{1}{2}\right]\frac{\pi}{2}\right) \left|\frac{\Gamma(\nu_s)}{\Gamma(\frac{3}{2})}\right|^8 \frac{(1 - \epsilon_V - s_V^S)^8 H^8}{Y_S^4 c_s^{12}}}{9\sqrt{3}k^3} \left\{ \frac{8\bar{\mathcal{U}}_1}{13312k^5} \left[ 65536\Gamma(17 - 8\nu_s)k^8 G_1^{equil} \right. \right. \\ - 16384i\Gamma(16 - 8\nu_s)k^7 G_2^{equil} + 4096\Gamma(15 - 8\nu_s)k^6 G_3^{equil} - 1024i\Gamma(14 - 8\nu_s)k^5 G_4^{equil} \\ + 256\Gamma(13 - 8\nu_s)k^4 G_5^{equil} - 64i\Gamma(12 - 8\nu_s)k^3 G_6^{equil} + 16\Gamma(11 - 8\nu_s)k^2 G_7^{equil} \\ - 4i\Gamma(10 - 8\nu_s)k G_8^{equil} + G_9^{equil} \left. \right] - \frac{\bar{\mathcal{U}}_2}{1024k} \bar{\mathcal{I}}^{equil} + \frac{838861\bar{\mathcal{U}}_3}{12} \left( \frac{\bar{\mathcal{Z}}_1^{equil} k^3 \Gamma(13 - 8\nu_s)}{67108864} \right. \\ \left. \left. + \frac{\bar{\mathcal{Z}}_2^{equil} k^2 \Gamma(14 - 8\nu_s)}{268435456} - \frac{\bar{\mathcal{Z}}_3^{equil} k \Gamma(15 - 8\nu_s)}{1073741824} - \frac{\bar{\mathcal{Z}}_4^{equil} \Gamma(16 - 8\nu_s)}{4294967296} + \frac{\bar{\mathcal{Z}}_5^{equil} \Gamma(17 - 8\nu_s)}{17179869184} \right) \right\} \quad (4.16)$$

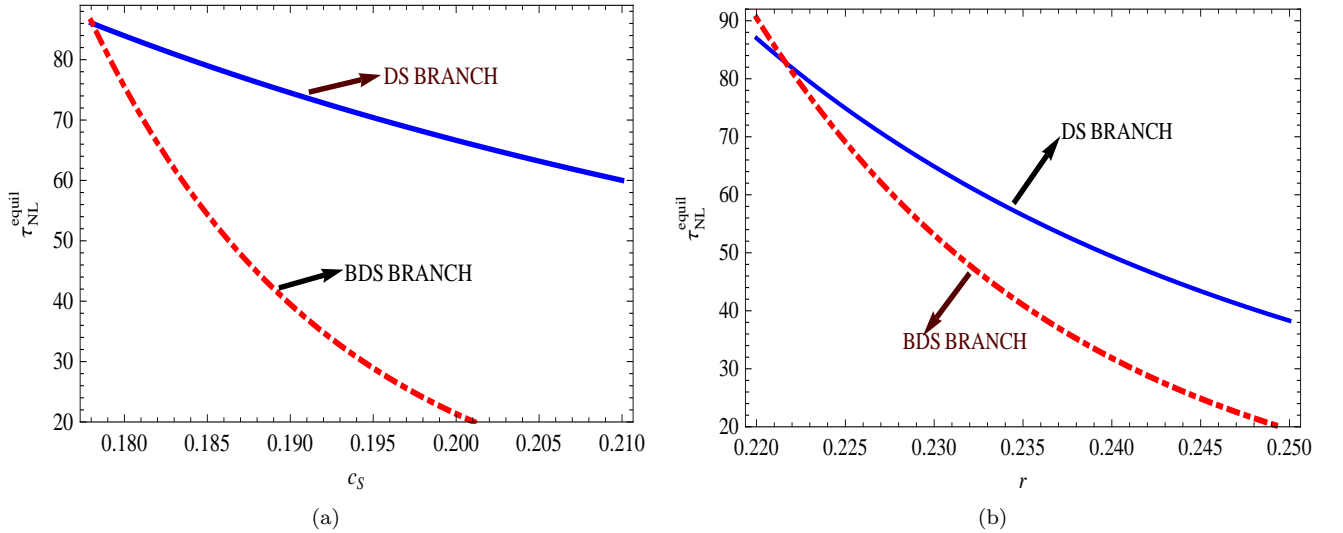


FIG. 2: (a) Variation of the  $\tau_{NL}$  vs sound speed for scalar modes  $c_s$ , (b) Variation of the  $\tau_{NL}$  vs tensor to scalar ratio  $r$  in *DS* and *BDS* limit.

In this context we have used  $\bar{\mathcal{I}}^{equil} = \bar{\mathcal{I}}(i, j; m, n)|_{i=j=m=n}(\forall i)$  and the similar argument holds for  $G_i^{equil}$  and  $\bar{\mathcal{Z}}_i^{equil} \forall i$ . The numerical value of  $\tau_{NL}^{equil}$  in the equilateral limit is obtained from our set up as  $38 < \tau_{NL}^{equil} < 89$  in *BDS* limit and  $48 < \tau_{NL}^{equil} < 97$  in *DS* limit within the window for tensor-to-scalar ratio  $0.213 < r < 0.250$  which is significantly large from other class of DBI models. Consequently this model can directly be confronted with the forthcoming observational data from PLANCK [2]. The graphical behavior of  $\tau_{NL}^{equil}$  with respect to the sound speed ( $c_s$ ) and tensor to scalar ratio ( $r$ ) are plotted in the figure(2)(a) and figure(2)(b) respectively. It is obvious from the figure(2) that  $\tau_{NL}^{equil}$  is not much sensitive to the tensor to scalar ratio ( $r$ ) or the sound speed ( $c_s$ ) within a specified range applicable for our model in *DS* and *BDS* limit analysis. In the next section, will show explicitly how this sensitivity problem is resolved in DBI Galileon.

## V. VIOLATION OF MALDACENA THEOREM AND SUYAMA-YAMAGUCHI RELATION

To show explicitly the violation of *Maldacena theorem*, we concentrate on squeezed limit configuration ( $k_i \simeq k_m \gg k_n$  with  $i, m, n = 1, 2, 3$  for  $i \neq m \neq n$ ) in which the non-Gaussian characteristics can be expressed as:

$$[f_{NL}^{sq}]_{DS} = \frac{5}{24c_s^2} \left[ (1 - n_\zeta) - \eta_s - 2\delta_{GX} - 3s_V + \frac{4\sqrt{V'(\phi)}M_{PL}^2}{\sqrt{\mathcal{G}(\phi)}}(\delta_{GX}\phi + 2\epsilon_V\epsilon_V') + \left\{ \frac{(1 - n_\zeta)}{2} - \frac{\eta_s}{2} - \frac{5s_V}{2} + \epsilon_V \right\} \delta_{GX} \right] \quad (5.1)$$

$$[f_{NL}^{sq}]_{BDS} = \frac{5}{3k_1^3} \left( \frac{k_3}{16k_1} \right)^{n_\zeta-1} \left| \frac{\Gamma(2 - \frac{n_\zeta}{2})}{\Gamma(\frac{3}{2})} \right|^2 \left\{ \left( 3 \left( 1 - \frac{1}{c_s^2} \right) - \frac{Y_S\delta_V}{c_s^2} + \frac{Y_S^2}{c_s^2} - \frac{2Y_Ss_V^S}{c_s^2} \right) \right. \\ \left[ \frac{3}{4}\mathcal{I}_1^{sq}(n_\zeta - 1) - \frac{3 - \epsilon_V}{4c_s^2} \left( \frac{1 + Y_S}{1 + \epsilon_V} \right)^2 \mathcal{I}_1^{sq}(\tilde{\nu}) \right] + \frac{3(1 - \epsilon_V - s_V^S)}{2Y_S} \left[ \mathcal{F}_3\mathcal{I}_3^{sq}(n_\zeta - 1) + \frac{\mathcal{E}_3}{c_s^2}\mathcal{I}_3^{sq}(\tilde{\nu}) \right] \\ - \frac{1}{8} \left[ \frac{Y_S}{2} + \frac{Y_S}{2} (3 - Y_S) \right] \mathcal{I}_4^{sq}(\tilde{\nu}) + \frac{Y_S}{4c_s^2} \left( \frac{4\epsilon_V - Y_S(3 - \epsilon_V)}{4(1 + \epsilon_V)} \right) \mathcal{I}_5^{sq}(\tilde{\nu}) + \frac{3(1 - \epsilon_V - s_V^S)^2}{Y_S} [\mathcal{F}_6\mathcal{I}_6^{sq}(n_\zeta - 1) \\ + \frac{\mathcal{E}_6}{c_s^2}\mathcal{I}_6^{sq}(\tilde{\nu})] - \frac{(1 - \epsilon_V - s_V^S)^2(1 + Y_S)^2(Y_S - \epsilon_V)}{2Y_Sc_s^2(1 + \epsilon_V)^3} \mathcal{I}_7^{sq}(\tilde{\nu}) + \frac{(1 + Y_S)(Y_S - \epsilon_V)(1 - \epsilon_V - s_V^S)}{4c_s^2(1 + \epsilon_V)^2} \mathcal{I}_8^{sq}(\tilde{\nu}) \left. \right\} \quad (5.2)$$

Consequently the three point correlation function in the squeezed limit can be expressed as:

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \rangle^{sq} = \frac{6}{5}(2\pi)^3\delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)f_{NL}^{sq}P(k_i)P(k_n) \quad (5.3)$$

which, in *DS* limit, turns out to be

$$\left[ \langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \rangle^{sq} \right]_{DS} = \frac{1}{4c_s^2} \left[ \left[ \langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \rangle_M^{sq} \right]_{DS} \left( 1 + \frac{\delta_{GX}}{2} \right) - (2\pi)^3\delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)P(k_i)P(k_n) (\eta_s + 2\delta_{GX} + 3s_V - \frac{4\sqrt{V'(\phi)}M_{PL}^2}{\sqrt{\mathcal{G}(\phi)}}(\delta_{GX}\phi + 2\epsilon_V\epsilon_V') - \left[ \frac{\eta_s}{2} + \frac{5s_V}{2} - \epsilon_V \right] \delta_{GX}) \right] \quad (5.4)$$

and in *BDS* limit it looks

$$\left[ \langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \rangle^{sq} \right]_{BDS} = \left[ \langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \rangle_M^{sq} \right]_{BDS} \times \left[ \frac{2}{k_1^3(1 - n_\zeta)} \left( \frac{k_3}{16k_1} \right)^{n_\zeta-1} \left| \frac{\Gamma(2 - \frac{n_\zeta}{2})}{\Gamma(\frac{3}{2})} \right|^2 \left\{ \left( 3 \left( 1 - \frac{1}{c_s^2} \right) - \frac{Y_S\delta_V}{c_s^2} + \frac{Y_S^2}{c_s^2} - \frac{2Y_Ss_V^S}{c_s^2} \right) \right. \right. \\ \times \left[ \frac{3}{4}\mathcal{I}_1^{sq}(n_\zeta - 1) - \frac{3 - \epsilon_V}{4c_s^2} \left( \frac{1 + Y_S}{1 + \epsilon_V} \right)^2 \mathcal{I}_1^{sq}(\tilde{\nu}) \right] + \frac{3(1 - \epsilon_V - s_V^S)}{2Y_S} \left[ \mathcal{F}_3\mathcal{I}_3^{sq}(n_\zeta - 1) + \frac{\mathcal{E}_3}{c_s^2}\mathcal{I}_3^{sq}(\tilde{\nu}) \right] \\ - \frac{1}{8} \left[ \frac{Y_S}{2} + \frac{Y_S}{2} (3 - Y_S) \right] \mathcal{I}_4^{sq}(\tilde{\nu}) + \frac{Y_S}{4c_s^2} \left( \frac{4\epsilon_V - Y_S(3 - \epsilon_V)}{4(1 + \epsilon_V)} \right) \mathcal{I}_5^{sq}(\tilde{\nu}) + \frac{3(1 - \epsilon_V - s_V^S)^2}{Y_S} [\mathcal{F}_6\mathcal{I}_6^{sq}(n_\zeta - 1) \\ + \frac{\mathcal{E}_6}{c_s^2}\mathcal{I}_6^{sq}(\tilde{\nu})] - \frac{(1 - \epsilon_V - s_V^S)^2(1 + Y_S)^2(Y_S - \epsilon_V)}{2Y_Sc_s^2(1 + \epsilon_V)^3} \mathcal{I}_7^{sq}(\tilde{\nu}) + \frac{(1 + Y_S)(Y_S - \epsilon_V)(1 - \epsilon_V - s_V^S)}{4c_s^2(1 + \epsilon_V)^2} \mathcal{I}_8^{sq}(\tilde{\nu}) \left. \right\} \left. \right] \quad (5.5)$$

Additionally we have used in *BDS* limit the following expressions:

$$\mathcal{I}_1^{sq} = \text{Cos} \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) k_1^3 \Gamma(1 + x) \left[ \frac{2+x}{2} - \frac{(1+x)}{2} \right] \mathcal{I}_2^{sq} = \text{Cos} \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \Gamma(1 + x) \left[ \frac{2k_1}{1-x} - \frac{k_1}{2} - \frac{1+x}{4} k_3 \right], \\ \mathcal{I}_3^{sq} = \frac{(k_1 k_3)^3}{8} \frac{\Gamma(3+x)}{2} \text{Cos} \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right), \mathcal{I}_6^{sq} = \frac{k_1 k_3^2}{8} \text{Cos} \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \frac{(6+x)\Gamma(3+x)}{12}, \\ \mathcal{I}_4^{sq} = \text{Cos} \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \left\{ \frac{k_1 k_3^2}{2} \left[ (3+x)\Gamma(1+x) - \Gamma(2+x) \frac{k_3}{2k_1} \right] + (\vec{k}_1 \cdot \vec{k}_3) k_1 \left[ (3+x)\Gamma(1+x) - \Gamma(2+x) \frac{1}{2} \right] \right\}, \\ \mathcal{I}_5^{sq} = \text{Cos} \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \left\{ \frac{k_1 k_3^2}{2} \left[ \Gamma(1+x) + \Gamma(2+x) \frac{k_3}{2k_1} \right] + \frac{(k_1 \cdot \vec{k}_3) k_1^2}{k_1} \left[ \Gamma(1+x) + \Gamma(2+x) \frac{k_1}{K} \right] \right\}, \\ \mathcal{I}_7^{sq} = \text{Cos} \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \frac{2+x}{2} \left[ \Gamma(1+x) + \Gamma(2+x) \left( \frac{1}{4} + (3+x) \frac{k_3}{8k_1} \right) \right] \left\{ \frac{k_1 k_3^2}{2} + (\vec{k}_1 \cdot \vec{k}_3) k_1 \right\}, \\ \mathcal{I}_8^{sq} = \text{Cos} \left( \left[ x - \frac{1}{2} \right] \frac{\pi}{2} \right) \left\{ \frac{k_1 k_3^2}{2} \left[ (3+x)\Gamma(1+x) + (3+x)\Gamma(2+x) \frac{k_3}{2k_1} - \Gamma(3+x) \frac{k_3^2}{4k_1^2} \right] \right. \\ \left. + (\vec{k}_1 \cdot \vec{k}_3) k_1 \left[ (3+x)\Gamma(1+x) + (3+x)\Gamma(2+x) \frac{1}{2} - \Gamma(3+x) \frac{1}{4} \right] \right\}. \quad (5.6)$$

Here

$$\left[ \langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3) \rangle_M^{sq} \right]_{DS} = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \underline{(1 - n_\zeta^{DS})} P_{DS}(k_i) P_{DS}(k_n) \quad (5.7)$$

$$\left[ \langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3) \rangle_M^{sq} \right]_{BDS} = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \underline{(1 - n_\zeta^{BDS})} P_{BDS}(k_i) P_{BDS}(k_n) \quad (5.8)$$

are collectively the well known *Maldacena theorem* which is widely accepted almost as a *no-go theorem* in calculating non-Gaussianity. However our result shows significant deviation from *Maldacena theorem*. Thus the most appealing and strong feature of our model coming from equation(5.3) is that the violation of well known *Maldacena theorem* at the tree level of non-Gaussianity. This essentially means one can have large non-Gaussianity even from a single field model of DBI Galileon. This feature is demonstrated by the non-linearities in the plots in figure(3). This directly confirms the credentials of DBI Galileon as the old DBI models fail to give rise to this result.

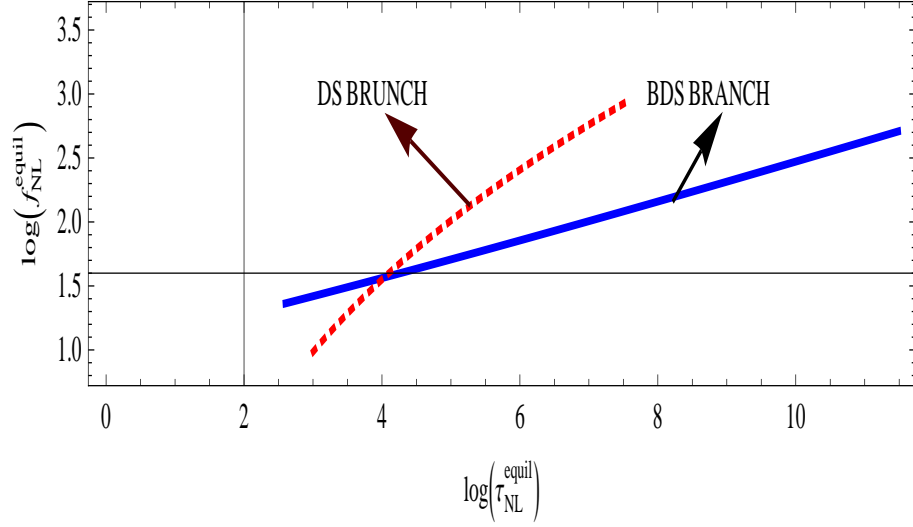


FIG. 3: Variation of  $\ln(f_{NL})$  vs  $\ln(\tau_{NL})$  in the equilateral limit configuration in *DS* and *BDS* limit.

Next using equation(3.9), equation(3.9), equation(4.10) and equation(4.11) in *DS* and *BDS* limit we get:

$$\begin{aligned} I_1^{DS} &:= \lim_{q \rightarrow 0} \int_{\vec{k}_1} \frac{d^3 k_1}{(2\pi)^3} \int_{\vec{k}_3} \frac{d^3 k_3}{(2\pi)^3} \langle \zeta(\vec{k}_1) \zeta(\underbrace{\vec{q} - \vec{k}_1}_{\vec{k}_2}) \zeta(\vec{k}_3) \zeta(\underbrace{-\vec{q} - \vec{k}_3}_{\vec{k}_4}) \rangle_{DS} \\ &= \lim_{q \rightarrow 0} \frac{2}{\pi} \int_{k_1=k_1^{IR}}^{\infty} k_1^2 dk_1 \int_{k_3=k_3^{IR}}^{\infty} k_3^2 dk_3 \delta^{(3)}(-\vec{k}_3) \mathcal{T}_\zeta^{DS}(\vec{k}_1, \vec{q} - \vec{k}_2, \vec{k}_3, -\vec{q} - \vec{k}_3), \end{aligned} \quad (5.9)$$

$$\begin{aligned} I_1^{BDS} &:= \lim_{q \rightarrow 0} \int_{\vec{k}_1} \frac{d^3 k_1}{(2\pi)^3} \int_{\vec{k}_3} \frac{d^3 k_3}{(2\pi)^3} \langle \zeta(\vec{k}_1) \zeta(\underbrace{\vec{q} - \vec{k}_1}_{\vec{k}_2}) \zeta(\vec{k}_3) \zeta(\underbrace{-\vec{q} - \vec{k}_3}_{\vec{k}_4}) \rangle_{BDS} \\ &= \lim_{q \rightarrow 0} \frac{2}{\pi} \int_{k_1=k_1^{IR}}^{\infty} k_1^2 dk_1 \int_{k_3=k_3^{IR}}^{\infty} k_3^2 dk_3 \delta^{(3)}(-\vec{k}_3) \mathcal{T}_\zeta^{BDS}(\vec{k}_1, \vec{q} - \vec{k}_2, \vec{k}_3, -\vec{q} - \vec{k}_3), \end{aligned} \quad (5.10)$$

$$I_2^{DS} := \lim_{q \rightarrow 0} \frac{\left| \int_{\vec{k}_2} \frac{d^3 k_2}{(2\pi)^3} \langle \zeta(\underbrace{\vec{k}}_{\vec{k}_1}) \zeta(\vec{k}_2) \zeta(\underbrace{-\vec{q} - \vec{k}_2}_{\vec{k}_3}) \rangle_{DS} \right|^2}{P_\zeta^{DS}(q)} = 16\pi^2 \lim_{q \rightarrow 0} \frac{\left| \int_{k_2=k_2^{IR}}^{\infty} k_2^2 dk_2 \mathcal{B}_{\zeta\zeta\zeta}^{DS}(\vec{q}, \vec{k}_2, -\vec{q} - \vec{k}_2) \right|^2}{P_\zeta^{DS}(q)}, \quad (5.11)$$



$$I_2^{BDS} := \lim_{q \rightarrow 0} \frac{\left| \int_{\vec{k}_2} \frac{d^3 k_2}{(2\pi)^3} \underbrace{\langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(-\vec{q} - \vec{k}_2) \rangle}_{\vec{k}_3} \right|_{BDS}}{P_\zeta^{BDS}(q)} = 16\pi^2 \lim_{q \rightarrow 0} \frac{\left| \int_{k_2=k_2^{IR}}^\infty k_2^2 dk_2 \mathcal{B}_{\zeta\zeta\zeta}^{BDS}(\vec{q}, \vec{k}_2, -\vec{q} - \vec{k}_2) \right|^2}{P_\zeta^{BDS}(q)}, \quad (5.12)$$

where we use the low energy IR momentum cut-off in computing the integrals. This leads to

$$\begin{aligned} I_1^{DS} &> I_2^{DS}, \\ I_1^{BDS} &> I_2^{BDS}. \end{aligned} \quad (5.13)$$

Substituting the explicit form of the bispectrum and trispectrum in equation(5.13) we get

$$\lim_{q \rightarrow 0} \int_{k_1=k_1^{IR}}^\infty k_1^2 dk_1 \int_{k_2=k_2^{IR}}^\infty k_2^2 dk_2 P_\zeta^{DS}(k_1) P_\zeta^{DS}(k_2) \left\{ [\tau_{NL}]_{DS} - \frac{36}{25} ([f_{NL}]_{DS})^2 \right\} > 0 \Rightarrow \boxed{[\tau_{NL}]_{DS} > \frac{36}{25} ([f_{NL}]_{DS})^2}. \quad (5.14)$$

$$\lim_{q \rightarrow 0} \int_{k_1=k_1^{IR}}^\infty k_1^2 dk_1 \int_{k_2=k_2^{IR}}^\infty k_2^2 dk_2 P_\zeta^{BDS}(k_1) P_\zeta^{BDS}(k_2) \left\{ [\tau_{NL}]_{BDS} - \frac{36}{25} ([f_{NL}]_{BDS})^2 \right\} > 0 \Rightarrow \boxed{[\tau_{NL}]_{BDS} > \frac{36}{25} ([f_{NL}]_{BDS})^2}. \quad (5.15)$$

This relation directly confirms the violation of *standard Suyama-Yamaguchi relation* in the context of single field *D3 DBI Galileon model* in SUGRA background thereby solving the generic sensitivity problem of DBI inflation. The above results clearly show that the so-called sensitivity problem is being resolved due to the violation of the Maldacena Theorem as well as the Suyama-Yamaguchi relation. The most probable reason for such violations are the appearance of non-canonical kinetic terms, its derivatives and non-standard stringy frame functions of DBI Galileon and Gauss-Bonnet correction in the SUGRA  $AdS_5 \otimes S^5$  bulk geometry. Such contributions directly affect the coefficients sitting in front of each and every term in the third and fourth order perturbative action. This results in the violation of standard results in non-Gaussianity which can solve the well known *sensitivity* problem in the context of DBI inflation. Very recently other novel aspects of the violation of well known consistency relations in the context of single field inflation has been studied in [33].

In figure(3) we have shown the parameter space for the non-Gaussian parameters in the logarithmic scale for equilateral limit configuration. More technical details can be found in the Appendix.

## VI. SUMMARY AND OUTLOOK

In this article we have elaborately studied the primordial non-Gaussian features from our proposed model of DBI Galileon inflation in D3 brane. We have derived the expressions for three and four point correlation functions in terms of the non-linear parameters  $f_{NL}$  and  $\tau_{NL}$  for local and equilateral type of non-Gaussian configurations in the nontrivial polarization modes. Hence we have explicitly demonstrated the violation of the well known *Maldacena theorem* and *Suyama-Yamaguchi relation* leading to a better observational bound for the above mentioned non-linear parameters as estimated from WMAP7 [1] dataset. Nevertheless, the results would fit better with the observational predictions of PLANCK [2] as expected to be available from their forthcoming datasets. The most significant outcome of our model is that it solves the so called *sensitivity problem* of DBI inflation by introducing Galileon degrees of freedom in effective D3 brane in SUGRA background within a specified range of parameter space. This directly confirms the physical importance of single field DBI Galileon over the other class of old DBI models as they fail to give rise to such promising result. These detectable features lead to the conclusion that this type of models can be directly confronted with upcoming data of PLANCK in near future in order to test the credentials of DBI Galileon.

The remaining open issues in the context of non-Gaussianity for DBI Galileon are studies of mass spectrum of primordial black hole formation [34], [35] as a tool for constraining non-Gaussianity at small scales, effect of the presence of one loop and two loop radiative corrections in the presence of all possible scalar and tensor mode fluctuations in the bispectrum and trispectrum, study of different shapes in equilateral, local, orthogonal, squeezed limit configuration for the tree, one and two loop level of non-Gaussianity and calculation of other higher order n-point correlation functions to find out the proper consistency relation between all higher order non-Gaussian parameters and the analysis of CMB bispectrum and trispectrum in the presence of Galileon in SUGRA background. Given the promise the results of the present paper shows, these open issues worth exploring in future as they may give rise to even more surprising results.

$[f_{NL;A}]^{u;(\lambda_1\lambda_2\lambda_3)}$ (E- mode)	DS $\times 10^{-3}$	BDS $\times 10^{-3}$	$[f_{NL;A}]^{u;(\lambda_1\lambda_2\lambda_3)}$ ( $E \otimes B$ - mode)	DS $\times 10^{-3}$	BDS $\times 10^{-3}$	$[f_{NL;A}]^{u;(\lambda_1\lambda_2\lambda_3)}$ (B- mode)	DS $\times 10^{-4}$	BDS $\times 10^{-4}$
$[f_{NL;1}]^{1;(000)} \text{ (PC)}$	2800 - 7000	4000 - 7000	$[f_{NL;1}]^{2;(000)} \text{ (PC)}$	0	0	$[f_{NL;1}]^{3;(000)} \text{ (PC)}$	0	0
$[f_{NL;2}]^{1;(0++)} \text{ (PV)}$	1.2 - 2.4	3.2 - 6.7	$[f_{NL;2}]^{2;(0++)} \text{ (PV)}$	1.0 - 2.3	2.1 - 4.5	$[f_{NL;2}]^{3;(0++)} \text{ (PV)}$	3.4 - 9.1	2.8 - 8.7
$[f_{NL;2}]^{1;(0--)} \text{ (PV)}$	0.1 - 1.8	1.4 - 5.7	$[f_{NL;2}]^{2;(0--)} \text{ (PV)}$	3.1 - 7.8	2.1 - 8.9	$[f_{NL;2}]^{3;(0--)} \text{ (PV)}$	3.4 - 8.2	2.7 - 7.2
$[f_{NL;2}]^{1;(0+-)} \text{ (PV)}$	1.7 - 6.2	2.6 - 9.6	$[f_{NL;2}]^{2;(0+-)} \text{ (PV)}$	3.1 - 6.7	2.9 - 11.0	$[f_{NL;2}]^{3;(0+-)} \text{ (PV)}$	1.4 - 10.2	2.7 - 8.4
$[f_{NL;2}]^{1;(0-+)} \text{ (PV)}$	4.1 - 9.3	1.7 - 6.9	$[f_{NL;2}]^{2;(0-+)} \text{ (PV)}$	2.2 - 8.3	3.5 - 7.4	$[f_{NL;2}]^{3;(0-+)} \text{ (PV)}$	1.1 - 9.8	1.8 - 10.6
$[f_{NL;3}]^{1;(00+)} \text{ (PC)}$	106 - 200	121 - 432	$[f_{NL;3}]^{2;(00+)} \text{ (PC)}$	90 - 245	78 - 349	$[f_{NL;3}]^{3;(00+)} \text{ (PC)}$	40 - 123	45 - 221
$[f_{NL;3}]^{1;(00-)} \text{ (PC)}$	445 - 992	549 - 878	$[f_{NL;3}]^{2;(00-)} \text{ (PC)}$	249 - 779	304 - 883	$[f_{NL;3}]^{3;(00-)} \text{ (PC)}$	123 - 621	189 - 588
$[f_{NL;4}]^{1;(+++)} \text{ (PV)}$	0.12 - 0.87	0.23 - 0.97	$[f_{NL;4}]^{2;(+++)} \text{ (PV)}$	0.09 - 0.24	0.08 - 0.32	$[f_{NL;4}]^{3;(+++)} \text{ (PV)}$	0.01 - 0.23	0.02 - 0.34
$[f_{NL;4}]^{1;(---)} \text{ (PV)}$	0.02 - 0.39	0.06 - 0.41	$[f_{NL;4}]^{2;(---)} \text{ (PV)}$	0.01 - 0.53	0.09 - 0.67	$[f_{NL;4}]^{3;(---)} \text{ (PV)}$	0.14 - 0.78	0.23 - 1.7
$[f_{NL;4}]^{1;(++-)} \text{ (PV)}$	0.19 - 0.89	0.23 - 0.93	$[f_{NL;4}]^{2;(++-)} \text{ (PV)}$	0.12 - 0.61	0.18 - 0.67	$[f_{NL;4}]^{3;(++-)} \text{ (PV)}$	0.09 - 0.44	0.03 - 0.53
$[f_{NL;4}]^{1;(+-+)} \text{ (PV)}$	0.04 - 0.21	0.01 - 0.35	$[f_{NL;4}]^{2;(+-+)} \text{ (PV)}$	0.06 - 0.32	0.07 - 0.44	$[f_{NL;4}]^{3;(+-+)} \text{ (PV)}$	0.05 - 0.33	0.02 - 0.42
$[f_{NL;4}]^{1;(-+-)} \text{ (PV)}$	0.01 - 0.34	0.04 - 0.39	$[f_{NL;4}]^{2;(-+-)} \text{ (PV)}$	0.01 - 0.21	0.02 - 0.32	$[f_{NL;4}]^{3;(-+-)} \text{ (PV)}$	0.08 - 0.48	0.09 - 0.51
$[f_{NL;4}]^{1;(-++)} \text{ (PV)}$	0.06 - 0.3	0.03 - 0.56	$[f_{NL;4}]^{2;(-++)} \text{ (PV)}$	0.09 - 0.49	0.1 - 0.43	$[f_{NL;4}]^{3;(-++)} \text{ (PV)}$	0.12 - 0.57	0.17 - 0.63
$[f_{NL;4}]^{1;(--+)} \text{ (PV)}$	0.06 - 0.29	0.09 - 0.34	$[f_{NL;4}]^{2;(--+)} \text{ (PV)}$	0.01 - 0.38	0.07 - 0.41	$[f_{NL;4}]^{3;(--+)} \text{ (PV)}$	0.04 - 0.32	0.05 - 0.44

TABLE I: Different non-Gaussian ( $[f_{NL;A}]^{u;(\lambda_1\lambda_2\lambda_3)}$ ) parameters related to the primordial bispectrum for A=1 (three scalar), 2(one scalar and two tensor), 3(two scalar and one tensor), 4(three tensor) with polarization index  $u = 1(E - mode)$ ,  $2(E \otimes B - mode)$ ,  $3(B - mode)$  including all helicity degrees of freedom represented by  $\lambda_1, \lambda_2$  and  $\lambda_3$  for DS and BDS limit estimated from our model. In this context “+”, “-” stands for two projections of helicity for graviton degrees of freedom and “0” represents helicity for scalar mode. Here **PC** and **PV** stands for the parity conserving and violating contributions appearing in the tree level primordial bispectrum analysis.

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### Appendix

Here throughout the computation we have used the following in-in oscillator modal identities

$$\begin{aligned} \langle 0|a(\vec{k}_1)a(\vec{k}_2)a(\vec{k}_3)a^\dagger(-\vec{k}_4)a^\dagger(-\vec{k}_5)a^\dagger(-\vec{k}_6)|0\rangle &= \langle 0|a(\vec{k}_4)a(\vec{k}_5)a(\vec{k}_6)a^\dagger(-\vec{k}_1)a^\dagger(-\vec{k}_2)a^\dagger(-\vec{k}_3)|0\rangle \\ &= (2\pi)^9 \left\{ \delta^{(3)}(\vec{k}_1 + \vec{k}_4) \left[ \delta^{(3)}(\vec{k}_5 + \vec{k}_2)\delta^{(3)}(\vec{k}_6 + \vec{k}_3) + \delta^{(3)}(\vec{k}_5 + \vec{k}_3)\delta^{(3)}(\vec{k}_6 + \vec{k}_2) \right] \right. \\ &\quad + \delta^{(3)}(\vec{k}_2 + \vec{k}_4) \left[ \delta^{(3)}(\vec{k}_5 + \vec{k}_1)\delta^{(3)}(\vec{k}_6 + \vec{k}_3) + \delta^{(3)}(\vec{k}_5 + \vec{k}_3)\delta^{(3)}(\vec{k}_6 + \vec{k}_1) \right] \\ &\quad \left. + \delta^{(3)}(\vec{k}_3 + \vec{k}_4) \left[ \delta^{(3)}(\vec{k}_5 + \vec{k}_1)\delta^{(3)}(\vec{k}_6 + \vec{k}_2) + \delta^{(3)}(\vec{k}_5 + \vec{k}_2)\delta^{(3)}(\vec{k}_6 + \vec{k}_1) \right] \right\}. \end{aligned} \quad (6.1)$$

$$\begin{aligned} \langle 0|a(\vec{k}_1)a(\vec{k}_2)a(\vec{k}_3)a(\vec{k}_4)a^\dagger(-\vec{k}_5)a^\dagger(-\vec{k}_6)a^\dagger(-\vec{k}_7)a^\dagger(-\vec{k}_8)|0\rangle &= \langle 0|a(\vec{k}_5)a(\vec{k}_6)a(\vec{k}_7)a(\vec{k}_8)a^\dagger(-\vec{k}_1)a^\dagger(-\vec{k}_2)a^\dagger(-\vec{k}_3)a^\dagger(-\vec{k}_4)|0\rangle \\ &= (2\pi)^{12} \left\{ \delta^{(3)}(\vec{k}_1 + \vec{k}_5) \left[ \delta^{(3)}(\vec{k}_2 + \vec{k}_6)\delta^{(3)}(\vec{k}_3 + \vec{k}_7)\delta^{(3)}(\vec{k}_4 + \vec{k}_8) + \delta^{(3)}(\vec{k}_2 + \vec{k}_7)\delta^{(3)}(\vec{k}_3 + \vec{k}_6)\delta^{(3)}(\vec{k}_4 + \vec{k}_8) \right. \right. \\ &\quad + \delta^{(3)}(\vec{k}_2 + \vec{k}_8)\delta^{(3)}(\vec{k}_3 + \vec{k}_7)\delta^{(3)}(\vec{k}_4 + \vec{k}_6) + \delta^{(3)}(\vec{k}_2 + \vec{k}_7)\delta^{(3)}(\vec{k}_3 + \vec{k}_8)\delta^{(3)}(\vec{k}_4 + \vec{k}_6) \\ &\quad \left. + \delta^{(3)}(\vec{k}_2 + \vec{k}_8)\delta^{(3)}(\vec{k}_3 + \vec{k}_6)\delta^{(3)}(\vec{k}_4 + \vec{k}_7) + \delta^{(3)}(\vec{k}_2 + \vec{k}_6)\delta^{(3)}(\vec{k}_3 + \vec{k}_8)\delta^{(3)}(\vec{k}_4 + \vec{k}_7) \right] \\ &\quad + \delta^{(3)}(\vec{k}_2 + \vec{k}_5) \left[ \delta^{(3)}(\vec{k}_1 + \vec{k}_6)\delta^{(3)}(\vec{k}_3 + \vec{k}_7)\delta^{(3)}(\vec{k}_4 + \vec{k}_8) + \delta^{(3)}(\vec{k}_1 + \vec{k}_6)\delta^{(3)}(\vec{k}_3 + \vec{k}_8)\delta^{(3)}(\vec{k}_4 + \vec{k}_7) \right. \\ &\quad + \delta^{(3)}(\vec{k}_3 + \vec{k}_6)\delta^{(3)}(\vec{k}_1 + \vec{k}_7)\delta^{(3)}(\vec{k}_4 + \vec{k}_8) + \delta^{(3)}(\vec{k}_3 + \vec{k}_6)\delta^{(3)}(\vec{k}_1 + \vec{k}_8)\delta^{(3)}(\vec{k}_4 + \vec{k}_7) \\ &\quad \left. + \delta^{(3)}(\vec{k}_4 + \vec{k}_6)\delta^{(3)}(\vec{k}_3 + \vec{k}_7)\delta^{(3)}(\vec{k}_1 + \vec{k}_8) + \delta^{(3)}(\vec{k}_4 + \vec{k}_6)\delta^{(3)}(\vec{k}_3 + \vec{k}_8)\delta^{(3)}(\vec{k}_1 + \vec{k}_7) \right] \\ &\quad + \delta^{(3)}(\vec{k}_3 + \vec{k}_5) \left[ \delta^{(3)}(\vec{k}_1 + \vec{k}_6)\delta^{(3)}(\vec{k}_2 + \vec{k}_7)\delta^{(3)}(\vec{k}_4 + \vec{k}_8) + \delta^{(3)}(\vec{k}_1 + \vec{k}_6)\delta^{(3)}(\vec{k}_2 + \vec{k}_8)\delta^{(3)}(\vec{k}_4 + \vec{k}_7) \right. \\ &\quad + \delta^{(3)}(\vec{k}_2 + \vec{k}_6)\delta^{(3)}(\vec{k}_1 + \vec{k}_8)\delta^{(3)}(\vec{k}_4 + \vec{k}_7) + \delta^{(3)}(\vec{k}_2 + \vec{k}_6)\delta^{(3)}(\vec{k}_1 + \vec{k}_7)\delta^{(3)}(\vec{k}_4 + \vec{k}_8) \\ &\quad \left. + \delta^{(3)}(\vec{k}_4 + \vec{k}_6)\delta^{(3)}(\vec{k}_1 + \vec{k}_8)\delta^{(3)}(\vec{k}_2 + \vec{k}_7) + \delta^{(3)}(\vec{k}_4 + \vec{k}_6)\delta^{(3)}(\vec{k}_1 + \vec{k}_7)\delta^{(3)}(\vec{k}_2 + \vec{k}_8) \right] \\ &\quad + \delta^{(3)}(\vec{k}_4 + \vec{k}_5) \left[ \delta^{(3)}(\vec{k}_1 + \vec{k}_6)\delta^{(3)}(\vec{k}_2 + \vec{k}_7)\delta^{(3)}(\vec{k}_3 + \vec{k}_8) + \delta^{(3)}(\vec{k}_1 + \vec{k}_6)\delta^{(3)}(\vec{k}_2 + \vec{k}_8)\delta^{(3)}(\vec{k}_3 + \vec{k}_7) \right. \\ &\quad + \delta^{(3)}(\vec{k}_2 + \vec{k}_6)\delta^{(3)}(\vec{k}_1 + \vec{k}_7)\delta^{(3)}(\vec{k}_3 + \vec{k}_8) + \delta^{(3)}(\vec{k}_2 + \vec{k}_6)\delta^{(3)}(\vec{k}_1 + \vec{k}_8)\delta^{(3)}(\vec{k}_3 + \vec{k}_7) \\ &\quad \left. + \delta^{(3)}(\vec{k}_3 + \vec{k}_6)\delta^{(3)}(\vec{k}_1 + \vec{k}_7)\delta^{(3)}(\vec{k}_2 + \vec{k}_8) + \delta^{(3)}(\vec{k}_3 + \vec{k}_6)\delta^{(3)}(\vec{k}_1 + \vec{k}_8)\delta^{(3)}(\vec{k}_2 + \vec{k}_7) \right] \left. \right\} \end{aligned} \quad (6.2)$$

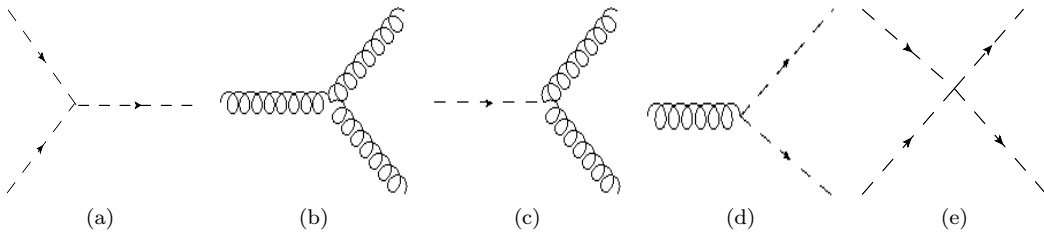


FIG. 4: Tree level Feynman Diagrams: (a)Three scalar correlator, (b) Three tensor correlator, (c)One scalar two tensor correlator , (d)One tensor two scalar correlator, (e)Four scalar correlator. In this context we have used spiral line for tensor contribution and dashed line for scalar contribution.

The tree level Feynman diagrams for three and four point correlation is drawn in figure(4).

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